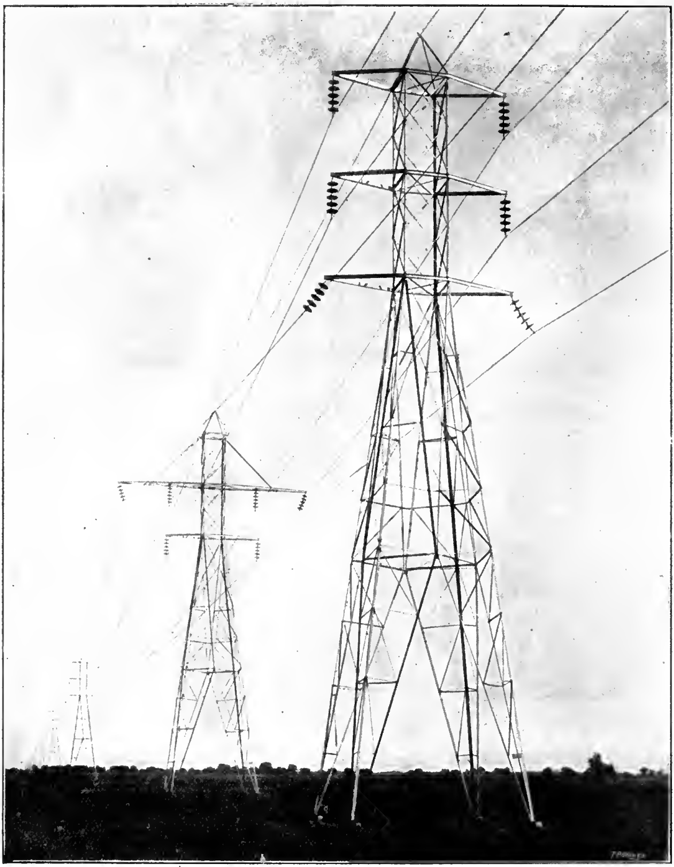
THREE-PHASE TRANSMISSION

W. Brew, M.I.E.E.

THREE-PHASE TRANSMISSION



Overhead Line for 100,000 Volts, showing Transposing Towers.

THREE-PHASE TRANSMISSION

A Practical Treatise

ON THE ECONOMIC CONDITIONS GOVERNING THE TRANSMISSION OF ELECTRIC ENERGY BY UNDERGROUND AND OVERHEAD CONDUCTORS

BY

WILLIAM BREW, M.I.E.E.

LATE CHIEF EXPERT-ASSISTANT, DUBLIN CORPORATION ELECTRICITY SUPPLY

NEW YORK
D. VAN NOSTRAND COMPANY
23 MURRAY AND 27 WARREN STREETS
LONDON
CROSBY LOCKWOOD AND SON
1911

PREFACE

WITH the increasing range of literature designed to meet the requirements of the electrical student, engineer, and specialist, some explanation may perhaps be offered for the appearance of a book devoting itself almost entirely to the electrical transmission of energy by three-phase currents.

That this system of transmission is eminently suited to modern requirements is obvious from the fact that, with the extended application of electricity to industrial purposes, undertakings formerly distributing single-phase or continuous current have alike resorted to three-phase transmission in order to enable them to cope with their widening field of operations. Thus with the very general use of three-phase transmission the practical consideration of the subject in all its bearings has become of the greatest importance.

Most engineers concerned with the generation and distribution of electrical energy have from time to time met difficulties involving financial and other considerations, which many text-books, excellent in other respects, have ignored completely, whereas the importance of sound financial principles in all technical questions cannot be overestimated.

It appeared, therefore, there was a want of a practical treatise upon the subject of three-phase transmission with definite aims in view. In the first place, to bring prominently before the reader such economical and financial points as engineers and specialists engaged upon new works would find useful; in the second place, to provide the earnest student with concrete examples of problems which, whilst demanding scientific treatment, are yet dependent upon commercial considerations for their useful solution.

Accordingly, in the following pages, the endeavour has been made to keep constantly in view the all-governing question:—Will it pay?

Whilst some knowledge of electrical engineering on the part of the reader is assumed, mathematics have been omitted as far as possible, and where algebraical expressions are introduced these are of the most elementary character.

References also have for the most part been omitted as uninteresting to the general reader and involving an amount of labour and research incommensurate with their utility to practical engineers.

The book contains some original investigation and much data not hitherto published, which, it is hoped, may prove of interest.

In conclusion, the author desires to express his indebtedness to Dr W. E. Sumpner for valuable suggestions, also to Messrs The British Insulated & Helsby Cables Ltd.; Ferranti Ltd.; Maschinenen-fabrik Oerlikon, Reyrolle & Co. Ltd.; and to the Council of the Institution of Electrical Engineers, for details and illustrations of plant kindly furnished by them. Much credit is also due to the publishers for the pains they have taken to make the book perfect in every respect.

W. B.

LONDON, *January* 1911.

CONTENTS

Contents

THREE-PHASE TRANSMISSION

CHAPTER I

INTRODUCTORY

IN the present work the writer proposes to discuss from the station engineer's standpoint the subject of three-phase power transmission, in connection with which huge sums of money have been sunk in the past, and much larger sums will probably be expended in the future with the natural growth of transmission schemes and the extended distribution of electricity for power, lighting, and traction purposes.

The trunk mains of the future within the United Kingdom will, according to the present tendency, largely consist of E.H.P. three-phase armoured cables laid underground, although it is well known that an overground line can generally be constructed from about half to one-third of the capital this involves. We may, however, see trunk mains of overground type more extensively adopted in the future.

The conditions to be met by the electrical transmission and distribution of power vary considerably in different countries. In England the problem usually resolves itself into the supply of energy in bulk to numerous consumers within comparatively small areas densely populated and within which coal is abundant. Accordingly, the use of overhead transmission lines is somewhat restricted, and the distribution of energy has to be effected by means of more expensive underground cable systems.

With transmission schemes abroad in countries where coal is scarce, and districts are sparsely populated, it is often economical to transmit power over very great distances by means of overground lines between water-power sources of energy and towns

or cities within which the energy is utilised for tramways, lighting, or industrial purposes. Accordingly, the working pressures in use on Continental and American transmission schemes greatly exceed those in use anywhere within the British Islands. An inspection of the following tables, giving particulars of some British and foreign transmission schemes, will at once render this evident. It will be seen that whereas the highest working voltage generally in use in England is 20,000 volts, some Canadian and American schemes are using pressures as high as 110,000 volts. Moreover, whilst the maximum distance over which energy is at present transmitted in England does not generally exceed 20 or 30 miles, we find energy transmitted over distances of 300 miles abroad.

It is of interest to note, however, that both in the United Kingdom and abroad it is becoming common practice to link up a number of generating stations to the same network of transmission lines whether these consist of overhead wires or underground cables or a combination of both, and this has become a distinguishing characteristic of the Power Companies in England working under special Act of Parliament over large areas, as compared with the more numerous Companies and Municipal Authorities working under Provisional Orders within strictly confined areas.

The linking up of a number of power stations to the same network of mains in the North of England has enabled power running to waste in the form of gas from coke ovens, &c., to be utilised. Such waste heat stations are installed at a number of points, and these are linked up to the steam-driven stations, which only supply the deficit of power required to meet the demand from consumers at any time.

On similar lines abroad, water power and steam-driven auxiliary stations are linked up to the same network, the steam-driven stations being used during the shortage of water which occurs at definite periods of the year in the case of many rivers. An interesting example of such a system is to be found in the South of France where one company, the Societé Energie Electrique du Littoral Mediterranéen, supplies direct 256 townships and indirectly a further 83 townships including ten tramway systems.

The synchronising of the various generating stations on the same H.T. network and over great lengths of transmission lines

has presented no difficulty, the impedance of the line in fact being found beneficial in keeping down the amount of the synchronising current.

TABLE I.—OVERHEAD TRANSMISSION SCHEMES ABROAD.

Undertaking.	Maximum Transmission Voltage.	Maximum Load in Kw.	Maximum Length of Transmission, Miles.
Hydro-Electric Power Commission, Canada	110,000	...	310
Spanish Hydro-Electric Co.	66,000	25,000	160
Spokane, Washington -	60,000	8,000	100
La Plombière, France -	57,000	3,000	124
Californian Gas and Electric Corporation	55,000	...	154 700 total
Gaucin, Seville - - -	52,000	3,900	78
Vizzola Campocologno, Northern Italy	45,000	36,000	113

TABLE II.—UNDERGROUND AND OVERHEAD TRANSMISSION SCHEMES—BRITISH.

Undertaking.	Maximum Transmission Voltage.	Maximum Load in Kw.	Maximum Length of Transmission, Miles.	Remarks.
Newcastle-upon-Tyne Electric Supply Co.	20,000	37,880	15	... ...
Cleveland and Durham Electric Power Co.	20,000	24,400	15	... ...
County of Durham Electric Power Co.	20,000	23,370	15	... ...
Clyde Valley Electric Power Co.	11,000	19,000	32	20 miles overhead.
North Metropolitan Electric Power Supply Co.	11,000	10,000	16	... ...
Yorkshire Electric Power Co.	10,000	10,000	21	... ...
South Wales Electric Power Distribution Co.	11,000	6,150	25	90 miles underground; 9 miles overhead.
Lancashire Electric Power Co.	10,000	5,300	12	60 miles underground; 14 miles overhead.

The extended use of E.H.P. underground cables involves some special considerations of an electrical, financial, and legal nature, regarding which little appears to have been written up to the present, and a brief review here of some of these may prove of interest to practical engineers.

Electrically, we have questions of the most suitable voltage for transmission with these cables under various conditions; the possible economies to be effected in working as regards copper, dielectric, and sheath losses; some sorely needed reforms in controlling switchgear; and methods of insulating at cable receivers, switches, and instrument connections; and quite a number of other points.

Financially, we have had before us in the past the phenomenally high price of copper and the possibility of this high price again holding, combined with a dear money market.

Legally, we have the requirements of the Board of Trade, the Home Office statutory obligations, and, in the case of municipal undertakings, the requirements of the Local Government Board also to meet.

In contemplating any extensions to plant or mains, the engineer has usually one or more of the following considerations before him :—

1. The urgency of the work necessary to cope with increasing business or of maintaining the continuity of the supply.

2. The limits of the extension advisable to be taken in hand immediately as influenced by the rate of growth of business on the one hand and the favourable or otherwise condition of the metal and money markets on the other hand.

3. Economies to be effected by the substitution of modern and efficient plant for obsolete plant, and the financial soundness of the change as shown by the saving to be effected and its ability to meet the annual charges of both existing capital commitments and of the capital required.

4. The enthusiasm or push factor of the promoters of rival methods of illumination, traction, or power, and the inevitable disaster resulting from a sitting-still policy common to any commercial undertaking.

The scope of the present chapter will only permit of a brief discussion of items Nos. 2 and 3 above.

With regard to 1, however, it may be said that the statutory obligations of a Corporation or Power Supply Company may

demand the first steps being taken irrespective of financial or other considerations. An emergency may require the engineer to decide things quickly, and may not permit of the careful weighing up of all considerations which should influence his decision towards the best end being achieved. A valuable asset in such cases is undoubtedly engineering instinct, provided it is successful. Few committees or boards of directors are humane in the case of failure.

With regard to 2, it will be convenient for reference in what follows if we review briefly at this point some financial considerations generally affecting extensions to electricity undertakings.

Taking the case in which a Municipal Authority is the undertaker, it is to be noted that the Local Government Board in sanctioning loans for municipal trading apparently allow the following periods for their repayment, such periods being supposed to represent the life of the various sections of the plant. The corresponding depreciation has been added on the assumption that the annual instalments are invested at 3 per cent. compound interest :—

TABLE III.

	Life—Years.	Depreciation at 3 per cent. C.I.
Engines	15	5.37
Dynamos	20	3.74
Switchboards	25	2.74
Cables laid solid	25	2.74
Cables armoured	15	5.37
Transformers	15	5.37
Buildings	50	0.89
Land	60	0.44

Borrowing powers having been sanctioned by the Local Government Board with due regard to the existing obligations of the municipality and to the rates not proving an undue burden to the citizens, the Municipal Authority may obtain a loan from the Treasury or in the open market at current rate of interest.

The effect of the period for which a loan is granted, and the rate of interest at which it can be obtained upon the annual repayments required for every £100 borrowed, are shown by the following table :—

TABLE IV.

Years of Loan.	Annual Repayments Covering Interest and Sinking Fund.			
	At $3\frac{1}{2}$ per cent.	At 4 per cent.	At $4\frac{1}{2}$ per cent.	At 5 per cent.
10	12	12.3	12.6	13.0
15	8.7	9.0	9.3	9.65
20	7.0	7.35	7.65	8.0
25	6.07	6.42	6.77	7.12
30	5.4	5.75	6.1	6.5
35	5.0	5.37	5.74	6.15

It may be mentioned, however, that should the money market quotations be abnormally high at a time it is necessary to effect a loan, powers are sometimes given to extinguish the loan at the end of five years or other period by the raising of a further loan upon more advantageous terms; the new loan in such cases being granted for the remaining number of years covered by the original loan.

As an example of one of the considerations governing an extension may be mentioned the laying down of additional cables, when trenches are open, before they are actually wanted, which will often effect economy in trenching and reinstatement, and avoid the obstruction of busy thoroughfares a second time. It is only proper, however, that in avoiding such obstruction the cost of doing so should in every case be considered.

During 1905, with copper bars at £76 per ton and lead at £13.8 per ton, the cost of a 0.15 three-core 6,000 volt cable laid and jointed in cast-iron trough was approximately £1,180 per mile. In 1907, with copper bars at £122 per ton and lead at £22.5 per ton, the cost of the same cable laid and jointed in cast-iron troughs was approximately £1,560 per mile. The cost of trenching and reinstatement did not alter appreciably within this interval, and for a single or double trough trench 30 inches deep in first-class setts may be taken as costing approximately £528 per mile. That is to say, £528 per mile would have been saved in trenching and reinstatement if two cables had been laid in 1905 upon the supposition that the second cable would not be required until 1907, two years later. Had a Municipal Authority been the undertaker, interest and sinking fund in accordance with the usual practice would have

had to be provided upon the capital represented by the cost of the second cable during the two years it was entirely unproductive. In the case of a company, however, the charges to revenue during the two years considered would have been limited to depreciation alone.

In 1905, loans for twenty-five years were granted to Municipal Authorities at $3\frac{1}{2}$ per cent., corresponding to an annual payment for interest and sinking fund of approximately 6 per cent. of the capital borrowed. In the example before us, therefore, with an unproductive capital amounting to £1,180 per mile of cable sunk for two years, the charge to revenue would have amounted to £141.6. On the other hand we should have effected an economy of £528 on account of trenching and reinstatement, leaving a saving of £386.4 per mile.

The above example, however, does not take into account three other very important factors. These are:—

(*a*) The variation of the money market,
(*b*) The variation of the metal market,
(*c*) Depreciation,

during the period of two years considered. For instance, loans were obtainable on a twenty-five years' basis in 1907 at about $4\frac{1}{2}$ per cent.

The rise in the price of metals increased the price of the cable from £1,180 to £1,560 per mile during this period.

The depreciation, in the ordinary sense, of the cable was discounted by the enhanced value of the metals used in its construction.

Taking into account the variations which occurred during the two years considered, a Municipal Authority putting down an extra cable in 1905 would have made annual repayments for interest and sinking fund during the twenty-five years' life of the cable, amounting in all to about £1,770 per mile. Had it deferred putting down the cable till wanted in 1907 and raised a loan then for the purpose, the total repayments on account of same during its life of twenty-five years would have amounted in all to about £2,640, a difference of £870, which, added to the saving in trenchwork of £386.4, represents a total saving of £1,256.4, *i.e.*, more than the entire cost of the extra cable itself if laid in 1905.

The scope of the present chapter will not permit of carrying this point further, but the example given will show the import-

ance of considering the extensions of an undertaking side by side with the prices holding in the money and metal markets respectively.

With regard to item No. 3 and the replacement of obsolete or inefficient plant by other of modern and more economical type, Municipal Authorities are very much in the hands of the Local Government Board in this respect, who may or may not grant a fresh loan until the original loan obtained upon the obsolete plant has been paid off; and unless the electricity undertaking is in the fortunate position of having a reserve fund put by out of revenue to cover depreciation and obsolescence it may be saddled with inefficient plant until it has run its natural life and the full interest and sinking fund instalments have been paid.

As an example illustrating this point, we may take transformers. In 1902 the average price of a 100 k.w. oil-cooled E.H.P. transformer was approximately £100, and the average magnetising loss on open circuit of this size of transformer about 1,200 watts. In 1907, in spite of the increased value of copper, the average price for this size and type of transformer remained approximately the same, but the magnetising losses of some of the best types were as low as 450 watts and averaged 600 watts. A loan obtainable in 1902 at $3\frac{1}{2}$ per cent. interest would represent an annual charge to revenue of £8.7 per £100 borrowed on the basis of a fifteen years' life. On the other hand loans effected in 1907 at $4\frac{1}{2}$ per cent. would represent an annual charge to revenue of £9.3 per £100 borrowed. Now in order to arrive at a financial result we shall require to know the value of the magnetising units in each case.

Assuming the old transformers to be scrapped and new ones substituted the annual charges to revenue will stand as follows:—

100 Kw. Transformer, 1902.		100 Kw. Transformer, 1907.	
Interest and Sinking Fund - - - -	£8.7	Interest and Sinking Fund—	
		New Transformer - -	£9.3
		Old Transformer - -	8.7
	£8.7		£18

In the case considered it will be found that for the number of magnetising watts required by the new and old transformers, *i.e.*, 600 and 1,200 respectively, it will pay to entirely scrap the old

transformers if the cost per unit of magnetising energy exceeds 0.425d.

With regard to consideration No. 4, it will be unnecessary to discuss at length the various rivals to the applications of electricity in its various branches. The keen competition of high-pressure incandescent gas lamps is now being met with flame arc lamps and metallic filament glow lamps. The competition of petrol-driven vehicles must be met by efficiently operated electric tramways, and the competition of isolated steam, suction gas, and other plants by electric power supply undertakings designed and worked upon a thoroughly sound financial basis. The developments of each competitor must be carefully and closely watched and kept ahead of by the electrical undertaking in its business capacity, its extensions and developments in every direction.

This brings us to the consideration of a state of affairs which has sometimes arisen in recent years, and which may prove competition of a serious nature to an existing electricity undertaking, or may, on the other hand, prove of material assistance, according to circumstances, that is, the possibility of a supplementary electricity supply being given by a Power Company situated without an area already served by an existing electricity undertaking.

In the event of an undertaking serving a definite area finding it desirable from want of capital, space for extension, or other reasons to supplement the supply of energy to its existing cable system by purchasing energy from an outside source, it is apparent that to obtain the full benefit of the dual supplies they should be capable of being worked in parallel, if discontinuity is to be avoided whenever change-over operations become necessary. In view of the many uses to which electricity is now put, and which demand absolute continuity in the supply, such interruptions as would be involved by changing over operations without paralleling could generally not be tolerated upon an extensive system.

In considering the feasibility of parallel working between the Power Company's system and the city supply system, we are met with such considerations as the synchronising of the two systems, the maintenance and sharing of the load between them in due and proper proportion, and the combined regulation of pressure at the city end of the Power Company's line.

If we assume certain substations within the city area are allocated to the Power Company to deal with, it will probably first be necessary that the line pressure of the Power Company be transformed to a pressure corresponding to that adopted upon the existing high-pressure feeders in connection with the city substations, to render the dual supply available, and secondly, transformed within the substations to the correct pressure in ordinary use by the existing consumers. As to whether the Power Company can under these conditions give a supplementary supply upon the same basis as the City Authority will depend upon the relative cost per unit of electrical energy delivered to the distributing network by the City Authority and Power Company respectively.

Bearing in mind that areas allocated to the Power Company would in general be outlying districts, the load factor may possibly be of a low order. If due to a lighting load pure and simple, we may assume a load factor of, say, 13 to 14 per cent.

Now the all-day efficiency of transformers working upon an extensive private lighting system with a load factor of 13 per cent. was found to be 87 per cent., that is to say of the total number of units per annum reaching the city from the Power Company's station, 13 per cent. would be accounted for in iron and copper losses in the transformers if it was necessary to convert the supply to the working pressure of the existing substations. In addition to this loss there is, of course, the transmission loss to be taken into account. We have, therefore, also to determine what the annual loss in the line would amount to under the conditions of the load factor assumed.

With the lighting load curves referred to having an average load factor of 13 per cent. for summer and winter, it was found that the square root of the mean square value of the load current throughout the year was very closely a third, *i.e.*, $\frac{1}{3.22}$ of the maximum current in the same interval. Using this value we can now arrive at figures for annual transmission loss given the maximum load current and resistance of the line.

For example assume :—

Voltage of transmission = 20,000 volts.
Drop in line at full load = 8 per cent.

Then we get :—

Annual transmission loss - -	2.48 per cent.
Annual transformation loss - -	13.0 ,,
	15.48 ,,

In the above example it is obvious that the Power Company could profitably supply the City Undertaking upon its existing basis of cost per unit if their own costs per unit were more than 15 per cent. below those of the City Undertaking under the working conditions assumed.

There are so many technical points of difference between the transmission of power by overhead conductors and by underground cables, that it may be advisable to review some of the more important of these briefly at this stage before discussing them in closer detail later on where analogous considerations arise with underground cables. We may review these conveniently under the following headings :—

Working Pressure
Impedance and Capacity.
Maximum Economy.
Protection from Lightning.

Working Pressure.—Up to the beginning of 1908 line insulators of pin type were largely in use in America and on the Continent with working pressures limited to about 60,000 volts. About this time the suspension type of insulator came into use, consisting of porcelain discs about 10 in. in diameter suspended one below the other. The number of discs required in series depends upon the line pressure, each disc being nominally capable of resisting a pressure of from 25,000 to 30,000 volts (Fig. 1).

With this type of insulator, the pressure which can be used upon the transmission line is only limited by the formation of the "corona" or brush discharge from the wires, which occurs at what is called the critical voltage and depends upon the diameter of the wires, their distance apart, atmospheric conditions, &c.

The loss of power which occurs from atmospheric dispersion after the critical voltage is reached becomes very heavy, accordingly, this point has become of primary importance in the design of high-pressure transmission schemes.

With a line pressure of between 150,000 and 200,000 volts it would appear that the limit has been reached in the voltage which can be employed on overhead lines using bare conductors, unless the wires are of abnormally large diameter or some special insulating covering be applied to the conductors to prevent the formation of the "corona" and consequent heavy loss of power in transmission.

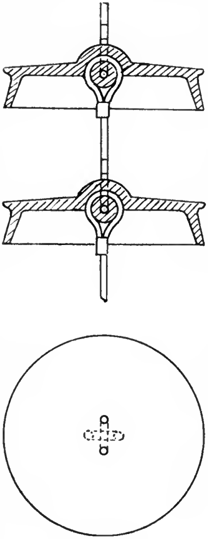


FIG. 1.

Impedance and Capacity. — The impedance of the overground line will be directly affected by the spacing of the wires and the frequency adopted with the system. The charging or capacity current of the line will be proportional to the working pressure, the frequency of the system, and will also depend upon the spacing of the line wires.

The spacing of the wires varies somewhat with different schemes. In the case of the 50,000 volt lines in the South of France the line wires are spaced 5 ft. 9 in. apart. In America spacings of 7 ft. and 10 ft. are common. A spacing of 10 ft. between wires would appear to be as great as can be efficiently adopted in most cases, owing to the fact that the self-induction of the line becomes of importance in increasing the reactance drop and reducing the power factor of the system. This will be seen from an inspection of the following table:—

TABLE V.—THREE 0000 S.W.G. WIRES CARRYING 100 AMPERES 100 MILES AT FREQUENCY=60 CYCLES.

Distance between Line Wires.	C.R. Drop per Line Wire.	Inductive Drop per Line Wire.	Total Drop between Line Wires.
Inches.	Volts.	Volts.	Volts.
36	3,367	6,560	12,730
48	3,367	7,020	13,500
70	3,367	7,430	14,130
120	3,367	8,180	15,350

The general practice is to run two lines of three wires each, one set on each side of a steel lattice work tower suspended by means of three cross arms. Typical arrangements are shown in Figs. 2 and 3.

The frequencies most usually adopted on the Continent are 25 and 50 cycles, and in America either 25 or 60 cycles. The use of the higher frequencies on long transmission lines is accompanied by largely increased reactive drop on the line, and also a proportionate increase in the charging current.

TABLE VI.—CHARGING CURRENT AT 60 CYCLES OF 100 MILES OF THREE-PHASE LINE 0000 S.W.G. WIRES.

Pressure between Line Wires.	Distance between Line Wires.	Charging Current in Amperes, per Line Wire.	Apparent Kilowatts.
Volts.	Inches.		
10,000	36	7.5	130
20,000	48	14.1	489
50,000	70	32.6	2,825
100,000	120	59.2	10,250

As the inductive drop upon the line will be proportional to the working current, this, as may be seen from Table V., is in the case of long transmission lines strictly limited, and will not generally exceed 100 amperes.

A further consideration limiting the working current is that of power surges or rises in pressure which occur when the line is broken or short circuited. The abnormal pressure rises encountered under such conditions are found to depend entirely upon the value of the current. It thus happens that a line working at 30,000 volts with a load current of 100 amperes may experience greater surge pressures than a line working at 60,000 volts with a load current of 50 amperes.

Maximum Economy.—The direct application of Kelvin's law to the transmission line gives the conditions under which the cost of transmitting a given amount of power along the line is a minimum. It does not, however, take into consideration the cost of insulating the line for extra high pressures, and this

becomes of much greater importance in the case of underground cables.

The usual condition applied to an overground transmission line is :—

$$\frac{\text{Total cost of transmitting power}}{\text{Revenue earned}} = \text{a minimum.}$$

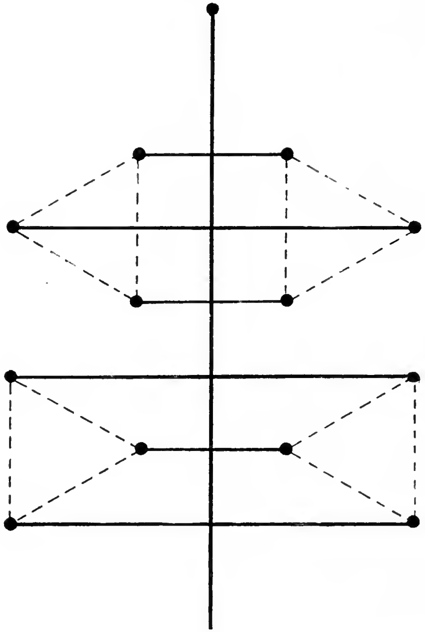


Fig. 2.

In this case the charges for interest and depreciation on the total cost of the line will be equal to the cost of the C^2R losses for maximum economy.

Under some conditions the drop in pressure along the line would be too great to enable regulation for constant pressure at the receiving end to be efficiently carried out if the most economical section of conductor were employed as given by

Kelvin's law. Under other conditions also the application of the law may be precluded by the excessive heating of the conductors.

Induction boosters placed at the distributing ends of transmission lines for compensating the line drop are adopted in some cases. This apparatus is, however, large and expensive, and it would appear that the more general practice with transmission schemes now is to vary the generator voltage with the load, keeping the line drop within the economical range of voltage regulation of the generator.

It is usual to keep the copper drop on the line down to

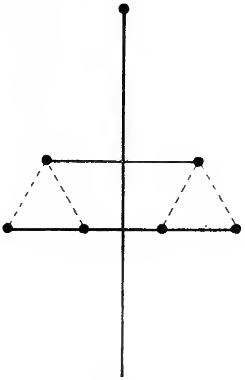


FIG. 3.

about 10 per cent., the total reactive drop at full load being about 15 per cent. This allows of regulation for approximately constant pressure at the receiving end of the line by the adjustment of the generator voltage.

The generators are mostly separately excited by independently driven exciters, and have a range of regulation in their voltage of as much as 25 per cent. in some cases.

It is interesting to note that the application of Kelvin's law in a modified form to many of the high-pressure transmission lines found abroad indicates that the very high working pressures adopted are in keeping with the conditions of maximum economy.

The conductors used for overground transmission are, in

general, constructed of cable consisting of seven or nineteen strands, which have greater flexibility than solid drawn conductors. The metals used in the construction of the line are either of copper or aluminium, steel being used, however, in very long spans. The relative advantages of copper and aluminium for this purpose may be briefly stated as follows :—

Copper.	Aluminium.
Greater mechanical strength.	Lighter than copper for same conductivity.
Easily soldered and jointed.	
Corrosion less likely than with aluminium.	Less expensive.
Smaller coefficient of expansion than aluminium, and, therefore, less sag.	Less danger of "corona" effect and loss from atmospheric dispersion.
Less cross section for the same resistance than aluminium, and, therefore, not affected to the same extent by wind pressure.	Less rise in temperature for the same resistance and working current than with copper.

Although aluminium has been extensively employed for overhead transmission lines, some of the later schemes are adopting stranded copper conductors.

Protection from Lightning.—The protection of the overhead transmission line from lightning discharges is of the greatest importance, although engineers in charge of high-pressure lines seem somewhat dubious as to the efficacy of most forms of lightning arrester. The lightning arresters in most general use are of electrolytic type arranged with a spark gap between them and the line.

As a further safeguard against lightning, an earthed wire is run the whole length of the line supported at the top of the transmission towers and situated about 6 ft. above the line conductors. Spark gap arresters are, in this case, generally also connected to the line.

The general experience is that transmission lines insulated for such high pressures as 100,000 volts appear to be much less affected by lightning than lines used at lower voltages.

CHAPTER II

TRANSMISSION LOSSES

AN account of the losses entailed by the transmission of electrical energy with underground cables of three-phase type, paper-insulated, lead-covered, and laid in cast-iron troughs or armoured with steel, must include the consideration of the following :—

a. Copper, or C^2R, loss due to the ohmic resistance R of the cores of the cable and the square of the load current C at every instant.

b. Dielectric hysteresis, or the loss due to mechanical stresses in, and the heating of, the insulation ; the specific inductive capacity of the insulation also allows a condenser current to pass and gives rise to a further C^2R loss in the cores of the cable.

c. Sheath loss due to currents induced in the lead sheath of the cable by the varying magnetic field produced by the alternating currents in the three cores of the cable.

d. Iron loss due to hysteresis proper, and eddy current loss due to the magnetisation of the steel armouring or cast-iron trough enclosing the cable.

With regard to the above it is first to be noted that losses *a*, *c*, and *d* depend upon the square of the load current, whereas loss *b* is independent of this load current, except in so far as it may increase the temperature of the cable and diminish the resistance of the dielectric.

Copper Losses.—As regards *a*, with a varying load curve such that c_1, c_2, &c., represent values of the load current during intervals t_1, t_2, &c., and R represents the resistance in ohms of one core of the cable, the total loss in units (with a three-core cable) in any given period is

$$\frac{3R(c_1^2t_1 + c_2^2t_2 + c_3^2t_3, \text{\&c.})}{1{,}000} \text{ units,}$$

where t_1, t_2, &c., indicate the intervals during which the load current had the corresponding values c_1, c_2, &c.

If we denote by Cm the average effective current and by T the total period considered, we have:—

$$C^2mT = c_1^2t_1 + c_2^2t_2 + \&c.,$$

or

$$Cm = \sqrt{\frac{c_1^2t_1 + c_2^2t_2 + \&c.}{T}}$$

That is, for any given load curve, the losses during the interval T would be the same as if the current had remained of constant value Cm, which is obviously the square root of the mean square of the current varying according to the load curve considered.

Dielectric Losses.—With regard to dielectric loss b it may perhaps be as well to consider briefly at this point the physical phenomena associated with insulating media subject to electrical pressure.

A .05 sq. in. three-core 20,000 volt cable, paper-insulated, has, when well constructed, an insulation resistance of about 1,000 megohms per mile at a temperature of 70° Fahr. This means, of course, a certain amount of leakage current, but as the loss per mile from this cause or the C^2R loss in the dielectric only amounts to 1.2 watts it is entirely negligible, upon the assumption of constant insulation resistance under electrical stress.

The capacity current of the same 20,000 volt cable, assuming a sine pressure wave free from harmonics at 50 cycles, would be approximately 0.74 ampere per mile, although in practice it might be much greater due to harmonics. The copper loss due to this current is given by C^2R (C being the charging current, and R the resistance of one core of the cable), and is only 0.4 watts per mile, and, therefore, for short cables also negligible. It might appear, therefore, that the really important loss must be looked for elsewhere, and was to be found in the so-called dielectric hysteresis loss of the cable. It will be shown, however, in what follows that the open circuit copper loss under certain conditions with long cables may considerably exceed the so-called dielectric loss.

When a tube of insulating material is subjected to a difference of electrical pressure between its inner and outer face, the material is electrostatically strained with a molecular displacement which

may finally end in the complete rupture of the material if the difference in pressure prove sufficient. If the pressure be removed, the molecular displacement would appear in most cases to gradually recover its normal unstrained condition, accompanied by an electrostatic phenomenon generally known as a soaking out of the charge. Whatever may be the real mechanism of this phenomenon the result remains that with rapidly alternating pressures molecular vibrations are set up in the dielectric, heating it and causing a loss of energy in a similar manner to that in which the rapid alternate magnetisation and demagnetisation of a piece of iron causes energy to be frittered away in the form of heat.

Associated with this loss due to molecular vibration in the dielectric is another effect, that is, the decrease in resistance of the dielectric as the time of its electrification is made shorter and shorter. This is well seen by the decrease in the deflection of a mirror galvanometer used to measure insulation resistance by the direct deflection method as the time of electrification or period of the test is extended, usually to an interval of one minute. Now with a cable subjected to an alternating E.M.F. it is obviously charged during one-quarter of a period, and hence the period of electrification is only one-fourth of the periodic time, and thus exceedingly short, *i.e.*, between $\frac{1}{100}$ second and $\frac{1}{400}$ second, with frequencies of 25 and 100 periods per second respectively. It would, therefore, appear that the resistance of a dielectric to an alternating pressure may be many times less than would be deduced from ordinary measurements of insulation resistance, and consequently the loss due to the dielectric acting as a conductor may be proportionately increased.

The charging current flowing into a cable due to capacity alone would have a phase difference of 90° in advance of the pressure, and would, therefore, be wattless. Owing to the copper resistance, however, of the conductors this phase difference is not exactly 90°, since the copper loss has to be supplied by means of a small power factor. Finally, the losses due to hysteresis and conduction in the dielectric must also be supplied by the charging current and pressure, and these losses, accordingly, increase the power factor of the cable to the necessary extent by bringing the pressure and charging current more nearly into phase.

It may be shown that, for the purpose of calculating the charging current which will flow into a three-phase cable with

symmetrical cores when subject to an applied pressure of sine wave form, we can assume that the three conductors themselves possess no capacity, but that they are each connected to the lead sheath of the cable by a condenser having an effective capacity which we may denote by K.

If we measure the capacity in microfarads between one core and the other two cores connected to the lead sheath and call this value K_1; also measure the capacity between all three cores bunched together and the lead sheath and call this value K_2, then the value of the effective condenser capacity we have denoted by K is given in microfarads by the following expression:—

$$K = 1.5K_1 - .166K_2.$$

It is well known that with a sine wave of effective pressure V volts and frequency n complete periods per second applied to the terminals of a condenser of capacity K microfarads, the charging current C in amperes is given by the expression:—

$$C = \frac{2\pi V n K.}{10^6}$$

We are, therefore, in a position to calculate the charging current of our three-phase cable.

Let us take as a practical example a modern three-phase .05 sq. in. paper-insulated lead-covered cable constructed for a working pressure of 20,000 volts between conductors, 10 miles in length, connected to a star-wound generator with earthed neutral point and developing a pure sine pressure wave.

Capacity measurements gave the following results:—

One core *versus* two other cores connected to lead sheath, *i.e.*, $K_1 =$ 1.7 microfarads.

All three cores bunched *versus* lead sheath, *i.e.*, $K_2 = 3$ microfarads.

Our effective condenser capacity, *i.e.*, K, is, therefore,

$$K = 1.5 \times 1.7 - .166 \times 3 = 2.06 \text{ microfarads.}$$

Now the effective pressure charging this condenser is $\frac{20,000}{\sqrt{3}}$ or 11,500 volts above earth potential.

Hence our charging current per conductor C is given by:—

$$C = \frac{2\pi \times 11,500 \times 50 \times 2.06}{10^6} = 7.45 \text{ amperes.}$$

It may be mentioned here that a .05 sq. in. three-core 20,000 volt paper-insulated cable with a dielectric of $\frac{1}{2}$ inch between cores and lead sheath, when pressure-tested at 40,000 volts and frequency of 84 cycles for four hours between one core and two others connected to lead sheath, was found to have increased in temperature by 36° Fahr. Similarly with a pressure of 30,000 volts the temperature rose 16° Fahr. in nine and a half hours. This temperature rise is nearly altogether attributable to dielectric loss, since the length of cable tested was a short one, and, therefore, the copper loss small. Wattmeter measurements showed the power factor of this cable to be about .028 with the wave form used in the test, and it would, therefore, appear that the dielectric loss with this cable in practice would be about .7 kw. per mile. As will be shown later, however, the power factor of any cable and the charging current will be largely influenced by the presence of harmonics in the wave form of the applied pressure.

Probably the earliest attempts to measure dielectric losses on practical cables were those made upon the Deptford 10,000 volt Ferranti mains by Mr D'Alton, when Chief Engineer to the City of London Company. The method of experiment consisted in carefully indicating an engine driving an alternator between such times as one cable after another was switched on or off. These measurements seemed to have corresponded with a power factor of .02 or a total loss of about 1 kw. per mile of cable. Various other methods have been adopted by other experimenters. Some of these results have been collected for reference in Table VII.

From these experimental results it is evident that we shall in general be fairly safe in assuming the power factor of a well-constructed paper cable to be about .028. The energy loss in watts W going on in the dielectric of a three-phase cable into which a charging current of C amperes is flowing under the effect of an applied pressure between conductors of V volts is, accordingly, given by :—

$$W = C.V. \sqrt{3} \times .028.$$

If the cable is a very long one, the C^2R loss due to the charging current C flowing through the conductors of the cable of resistance R will become of importance, and may considerably exceed the dielectric loss.

TABLE VII.

Authority.	Type of Cable Working Pressure and Frequency.	Power Factor Observed.	Method of Measurement.
D'Alton - -	Deptford Mains Ferranti C.C. paper-insulated cables, 10,000 volts, 87 ~	.02	Indicating engines while cables were switched on and off.
Sparks - -	V.I.R. C.C. cable, 2,000 volts, 100 ~	.034	D.C. motor-driven alternator.
Mather - -	V.I.R. C.C. cable 2,000 volts, 100 ~	.034	Wattmeter method. Cable alone, and in parallel, and series with ironless choker.
Ayrton and Mather	British Insulated and Helsby C.C. paper cable, 2,000 volts, 100 ~	.024-.028	Wattmeter method. Ironless choker in parallel and series with cable.
Do. - - -	Silvertown V.I.R. C.C. cable, 2,000 volts, 100 ~	.028	Do.
Mordey and Minshall	V.I.R. C.C. cable, 2,000 volts, 100 ~	.124	Thomson recording wattmeter.
Do. - - -	V.B. C.C. cable, 2,000 volts, 100 ~	.124	Motor-driven alternator. Swinburne wattmeter.
Hoor - -	Paper cable, 2,000 volts, 50 ~	.025	Wattmeter.
British Insulated and Helsby Cables Ltd.	Helsby rubber cables, C.C. 50 ~ and 100 ~, 2,000 volts.	.0228-.0255	Wattmeter and choker as in Ayrton's experiments.
Do. - - -	Three-core .05 sq. in. 20,000 volt paper cable	.028	Do.

To arrive at the value of this copper loss due to the charging current we must bear in mind that this current continuously diminishes in value as it travels along the cable from the sending end. In the case of any one core of a cable of resistance R ohms into which a charging current of C amperes is flowing at the sending end, it is readily shown that the total copper loss W in watts is given by :—

$$W = \tfrac{1}{3}C^2R,$$

or the equivalent current C_1 which would produce the same loss if uniformly distributed along the cable is

$$C_1 = \sqrt{\tfrac{1}{3}}C.$$

Taking all three cores of our three-phase cable into consideration, we get the total copper loss, C^2R.

Lead Sheath Losses.—The sheath losses in the case of a three-core cable will depend upon—

1. The distribution and variation in the magnetic field set up by the currents in the cable cores in air or other non-magnetic medium.

2. The extent to which this field is augmented by the cast-iron trough or steel armouring by which the cable is enveloped.

3. The ohmic resistance of the lead sheath to the currents induced in it by the E.M.F. produced by the varying magnetic field.

FIG. 4.

It has also been fairly well established experimentally that the following laws hold with regard to sheath loss :—

1. It is directly proportional to the length of the cable.

2. It increases as the square of the current in the cores.

3. It is very nearly proportional to the square of the frequency.

4. It is inversely proportional to the resistance of the lead sheath, and hence approximately proportional to its thickness with a given diameter over dielectric.

If we imagine the lead sheath divided into three segments, each situated symmetrically over one core of the cable, Fig. 4, the total current in each segment will be in quadrature with the current in the core adjacent to it, and its value may be approximately arrived at by calculation from the known relation between two mutually inductive circuits in air :—

$$I_2 = \frac{2\pi n M I_1}{I m p_2}.$$

To get a rough idea of the manner in which the E.M.F.'s are induced in the lead sheath we will assume that the current in each conductor rises to a maximum, decreases and passes through zero to a negative maximum in accordance with a sine law. If at time 0 we suppose the current in core No. 1, C_1, to be 100 amperes, the successive values of the currents C_1, C_2, C_3 at equal time intervals of 30° will be as shown by the following table:—

TABLE VIII.

Time.	C_1.	C_2.	C_3.
0	100	50	50
30	86	0	86
60	50	50	100
90	0	86	86
120	50	100	50
150	86	86	0
180	100	50	50

If we imagine the currents induced in the three segments of the sheath to have equal values to those in the three corresponding cores, their relative values are given by the first and fourth, second and fifth, &c., lines in the above table.

Now it is evident that although, owing to the symmetrical arrangement of the three cores of the cable, the variation in the distribution and strength of the field throughout a complete cycle may be calculated and plotted in a diagram, provided no magnetic material such as an iron envelope is in the neighbourhood of the cable; yet it is difficult to predict to what extent the field produced by the cable cores is augmented by iron envelopes such as cast-iron troughing or steel armour, and, further, in what manner the natural distribution of the field is interfered with.

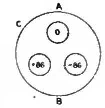
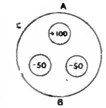
In order to test this point the writer prepared a search coil of rectangular form 22 cm. in length and 1 cm. in width, and composed of 124 turns of fine wire. The search coil was sufficiently narrow to allow of its being inserted in any position round the periphery of the cable, illustrated by Fig. 4 (the usual jute layer between armour and lead having been removed), or in a similar manner between the lead sheath and the cast-

iron troughing enclosing the cable. Direct currents were then passed through the three cores of the cable corresponding in value and sign to those at different instants throughout a complete period when working three-phase.

The search coil having been previously connected to a carefully standardised Ayrton & Mather Ballistic Galvanometer the throw of the needle when the direct current circuits through the cores were interrupted was carefully noted.

The following readings (Table IX.) illustrate the results obtained :—

TABLE IX.

DIAGRAM.	Particulars of Cable.	Position of Search Coil and Strength of Field in C.G.S.			Position of Search Coil and Number of Times Field is Greater than with Cable Unenclosed			Average Increase in Field at Three Positions, A, B, C.
		A	B	C	A	B	C	
	Unenclosed -	1.73	2.38	1.79	...	...	...	...
	In C.I. trough -	3.04	4.61	2.38	1.75	1.92	1.33	1.66
	Steel armour -	2.28	3.72	2.53	1.32	1.55	1.41	1.42
	Unenclosed -	1.19	.198	2.71	...	...	...	...
	In C.I. trough -	2.5	.228	4.6	2.08	1.11	1.7	1.63
	Steel armour -	1.68	.34	3.73	1.41	1.69	1.38	1.47

It might appear at first sight that the above values obtained for strength of field under various conditions are much smaller than would be expected from the large currents traversing the cores of the cable. The explanation is, however, not far to seek, and the discrepancy is due to the fact that as the plane of the coil was necessarily tangential to the circumference of the cable the direction of the lines of force was in no case normal to the plane of the test coil. Nevertheless a comparison of such readings, taken with as nearly as possible identical positions of

the search coil and the same current values first without and then with the iron envelope surrounding the cable, should still represent with close approximation the extent by which the external field of the cable is augmented by the troughing or armouring.

Coming now to the direct measurement of lead sheath losses upon underground cables, this is in practice usually accompanied by some difficulty arising from the following amongst other points.

(1) The loss to be measured is generally such a small fraction of the total load carried by the cable at high pressure that direct measurement by the ordinary switchboard wattmeters is out of the question.

(2) Measurement by currents at low potential are likely to be seriously affected by variation in the copper resistance of the cable from rise in temperature during the test, on account of the large currents usually necessary to reproduce working conditions of field, &c.

If the three cores of an underground high-pressure cable be joined together at the far end and three-phase current at low pressure be passed through the cable, from observations of the potential difference V in volts between the cores of the cable and the current C in amperes flowing into each core at the sending end, the impedance per core in ohms is given by

$$\frac{V}{\sqrt{3}\,C}.$$

If we measure also the copper resistance per core R in ohms the product CR gives us the effective E.M.F. (e).

Owing to the fact, however, that the lead sheath of the cable is acting as a closed secondary circuit to each core of the cable as a primary circuit, the power factor cos ϕ will not be given by the quotient $\frac{e\sqrt{3}}{V}$, since the effect of the closed secondary will be to bring the impressed pressure $\frac{V}{\sqrt{3}}$ and the effective pressure (e) more nearly into phase than is indicated by the value of ϕ so obtained. Oscillograph records of P.D. and current in the case of a cable of the size shown in Fig. 4 showed that ϕ was small, and hence cos ϕ was practically unity.

There is no doubt, however, that the lead sheath losses will lie between the values

$$C\left(\frac{V}{\sqrt{3}}-e\right) \text{ and } C\left(\frac{V}{\sqrt{3}}\cos\phi - e\right),$$

or the apparent watts less copper watts with assumed angles of lag zero and ϕ respectively.

It may, therefore, be of interest to ascertain the maximum values such losses could reach in practice.

In Table X. are set out observations made upon a length of 5,480 yds., *i.e.*, 3.11 miles, of this, 0.15 sq. in. three-core 6,000 volt cable, having a lead sheath 0.25 in. in thickness, and enclosed in a cast-iron trough $4\frac{1}{8}$ in. by $4\frac{1}{8}$ in. by $\frac{3}{8}$ in. in thickness, the full-size section of the cable being given by Fig. 4.

The resistance of each core was found to be 0.901 ohm.

TABLE X.

Amps. C	$\frac{V}{\sqrt{3}}$	$\frac{VC}{\sqrt{3}}$	C^2R	$\frac{VC}{\sqrt{3}} - C^2R$
24.5	25	612	541	71
30.8	30.5	940	853	87
35.4	35.2	1,246	1,127	119
34.1	34.3	1,175	1,048	122
31.25	31.25	975	880	95
25.5	25.6	652	587	65

From the above table, allowing for ordinary errors of observation in the readings of ammeters and voltmeters, it will be seen that the maximum loss as represented by the last column increases closely as the square of the current in the cable cores. Assuming the full load current of this cable to be 130 amperes per core, the maximum loss per mile with all three cores would be

$$\left(\frac{130}{34.1}\right)^2 \times \frac{122}{3.11} \times 3, \text{ say} = 1,710 \text{ watts.}$$

Now it was found that the effect of the cast-iron trough was to increase the strength of field surrounding the cable by 1.64 times, whereas steel armouring increased the strength of field by 1.44 times its value in air.

As the sheath losses will vary as the squares of these

numbers we arrive finally at the following maximum values of the lead sheath losses in the cable considered.

TABLE XI.—LOSS IN KW. PER MILE AT 130 AMPERES PER THREE CORES AT 50 ~.

Cable enclosed by C.I. Trough.	Cable enclosed by Steel Armour.	Cable unenclosed in Air.
1.71	1.320	.636

Board of Trade Regulations.

Having briefly reviewed the nature and order of the losses to be met with in underground E.H.P. three-phase mains, it is of interest to consider at this stage the influence of the Board of Trade Regulations upon transmission schemes employing these cables.

Regulation B, Clause 2, of the E.H.P. Regulations of the Board of Trade, dated 1906, reads as follows:—

"A main for an extra high-pressure supply shall not, without the consent in writing of the Board of Trade, be used for the transmission of more than 1,000 kilowatts unless adequate provision is made for an emergency supply in the event of the breakdown of the main."

Although the Board of Trade will, in accordance with their usual practice, consider each case upon its merits, and in general issue their consent in cases where the commercial aspect of the question would otherwise irretrievably hinder electrical progress, yet the result of the 1,000 kw. limit per cable if rigidly enforced would prove somewhat hard upon some electrical undertakings, as will be shown in what follows.

The first point which will be at once apparent is that a number of similar cables in parallel will be required according to the load to be transmitted to each point, and since on the grounds of safety the Board of Trade recommend that trunk feeders should, if possible, take different routes, the cost of trenchwork will in general be in proportion to the number of cables required. In addition extra expense is involved in the cost of cable. For instance, with a 20,000 volt transmission, two .025 three-core cables laid and jointed would cost £2,528 per

mile as compared with £1,544 per mile for a three-core .05 cable under similar conditions.

The second point to be considered is the effect upon the working losses if each cable be laid for transmitting 1,000 kw.

In considering this point, we must first settle the means to be adopted for maintaining approximately constant pressure at the receiving ends of the cables.

For the purpose of distributing electrical energy over any extended area, one or several generating stations, according to the nature of the problem, may be efficiently employed. There is, however, in every case the consideration of pressure regulation at the generating stations or in the substations to compensate for the drop in pressure in the transmission cables or the line loss. The large standard types of alternators used at the present time in power stations are not suitable for giving their output at widely different pressures. Since such machines are required to give their maximum pressure with maximum load they would of necessity have to be run much under-excited at times of light load when the line drop was small, by means of main field rheostats, that is, if the range of pressure variation assumed be greater than can be dealt with by a shunt regulated exciter. Under such conditions the regulation of pressure would be very unstable, the voltage creeping up or down after every adjustment of the rheostat, and every fluctuation in the load would be accompanied by wide variation in pressure. Quite apart from this, there is always present the necessity of keeping some circuits, if only local lighting circuits and those dealing with motor-driven auxiliaries, at approximately constant pressure.

The question of boosting the whole or portion of the output of the generating station must in every case be considered on its merits, and the working costs of the booster with the particular load curve to be met considered side by side with the interest charges on the capital cost of the extra copper, which if put into the line or cable system would render boosting unnecessary. The writer has met with cases where the double transformation of the load to enable boosting to be effected involved an annual cost of as much as £1 per kw. transmitted with a lighting load curve.

Where the station output is transmitted for lighting purposes, and it is necessary to supply various trunk mains of different length in which the peak of the load occurs at different times,

the adjustment of pressure at the generating station bus bars will not suffice to maintain constant pressure at the various points of distribution. If the trunk cables also transmit a motor load, it is not possible to avoid by regulation at the generating station variations in the pressure at the distributing points arising from the fluctuations in the load.

By the use of booster bus bars the pressure on one or more groups of trunk cables of approximately equal length may be adjusted simultaneously at the generating station or the regulation may be effected at the substation ends of each set of cables.

A common form of booster for such service consists of two parts, rotor and stator, as in a three-phase induction motor, the windings in the simplest case being connected in series. By means of a worm gear and hand-wheel (or automatically if desired) the rotor can be displaced relatively to the stator. We have, in fact, a static transformer in which the primary and secondary circuits are movable relatively to one another, and, according to the position of the rotor, relatively to the stator; the resultant pressure of the rotor or one of its components will add to or diminish the stator pressure.

Such induction boosters have the advantage of possessing a continuous range of regulation, and also allow of fine adjustment throughout their range, whereas boosting transformers with stops only give a limited number of fixed pressures, and regulating switches with such transformers are usually limited to working voltages of between 2,000 or 3,000 volts.

The pressure induced by the rotor windings will vary from zero to a maximum value positive or negative according to its position relatively to the stator.

Where the regulation is required to be effected on an E.H.T. circuit the rotor winding of the booster is usually fed by a transformer, permitting of the movable portion of the booster working at low pressure.

Fig. 5 illustrates the usual connections in such cases.

Some financial considerations governing the application of boosting appliances will be found set out in Chapter VII.

An alternative is to employ separate steam-driven exciting plant in combination with shunt or main field regulation to give the required variation in the pressure of the generator.

Another method of regulation consists in employing syn-

chronous motors at the receiving end of the line, and by varying the excitation of the synchronous motors compensating for inductive drop in the line and low power factor.

Each of the above boosting arrangements, however, involves :—

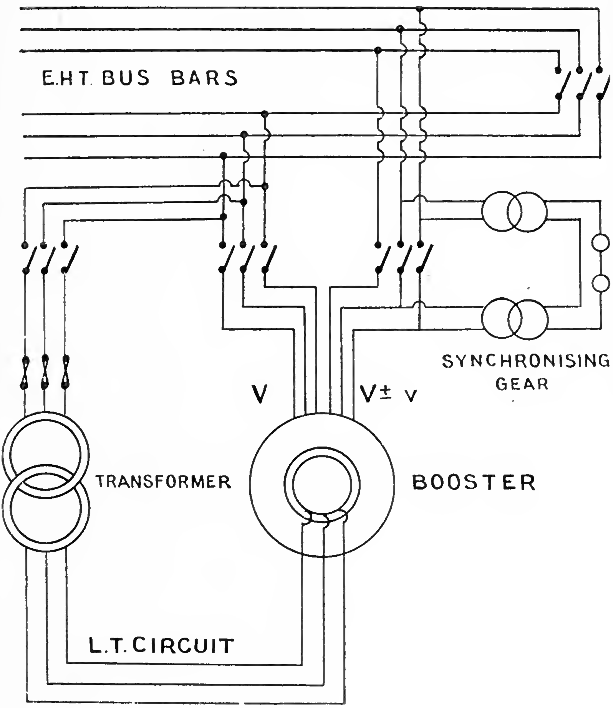
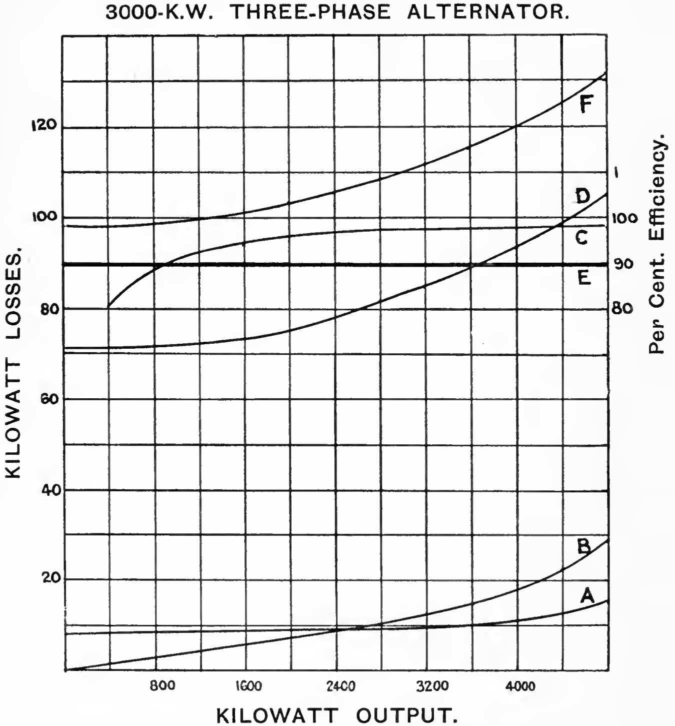


FIG. 5.

(1) Complication of switchgear and control.
(2) Increased liability to breakdown.
(3) Extra cost in capital and running charges.

In general with such E.H.P. distribution schemes as are likely to be adopted in the British Isles, the range of regulation obtainable by a direct-driven exciter upon the alternator shaft will be

found convenient and economical, any extra long feeders being regulated by boosters either at the generating station or at the substations. We may, therefore, discuss at some length this more general case.



A=Total Excitation Loss.
B=Armature Copper Loss.
C=Commercial Efficiency.
D=Total Electrical Losses.
E=Iron Wind and Friction Losses.
F=Total Losses.

FIG. 6.

With three-phase alternators of 1,000 to 3,000 kw. output, the excitation required by the main field will generally be

between 7 and 10 kw. (see Fig. 6) involving currents of 200 amperes and upwards in the main field, if the common practice of a low voltage exciter be followed ; with such machines regulation by rheostats in the main field requires the use of large resistances which are both costly in themselves and wasteful in operation the more economical method being the insertion of a regulating rheostat in the shunt winding of the exciter. This method of regulation has, however, the following characteristics :—

(1) At light loads when considerable resistance is inserted in the shunt winding of the exciter, regulation becomes somewhat unstable, due to the voltage of the exciter requiring some time to attain a steady value after operating the rheostat, and in addition the weak field of the alternator is likely to cause considerable variations in terminal pressure with even small fluctuations in the load.

(2) The total rise in the pressure of the alternator at full load is usually limited to about 10 per cent.

As it will be as well to keep a margin of not less than 5 per cent. in hand of the possible regulation to cope with irregularity in steam supply, emergency loads, &c., and an allowance of at least 3 per cent. at the receiving end of the line to make up the voltage drop on the transformers and distribution system of mains, it will be seen that a line loss of 2 per cent. or thereabouts would be convenient (if we exclude boosting apparatus) with this system of generating and transmitting at extra high pressure. No account has been taken of the hand regulation of the engine governor which would generally be available, since the resulting variation in the frequency entailed thereby should be discountenanced in ordinary working.

On the basis of a 2 per cent. line loss and the transmission of 1,000 kw. per cable, the following table gives the approximate sectional area and cost, laid and jointed, of three-phase extra high-pressure paper-insulated lead-covered cables armoured and suitable for delta working at various pressures over the distances stated. The prices of the cables in each case are based upon copper electrolytic wire bars at £120 per ton and lead at £20 per ton, the highest prices which have held in recent years. It will be hardly necessary to point out that had we assumed a different percentage line loss and other prices for copper and lead similar features would have been exhibited to those illustrated by the table under consideration.

TABLE XII.—APPROXIMATE SECTIONAL AREA AND COST OF THREE-PHASE TRUNK MAINS TRANSMITTING 1,000 K.W. WITH A LINE LOSS OF 2 PER CENT.

The Cables are insulated for Delta working.

Voltage of Transmission.	5 Miles.		10 Miles.		15 Miles.		20 Miles.		50 Miles.	
	Area per Core.	Cost per Mile.	Area per Core.	Cost per Mile.	Area per Core.	Cost per Mile.	Area per Core.	Cost per Mile.	Area per Core.	Cost per Mile.
		£		£		£		£		£
5,000	.40	2,928	...	...	...	...	...	...	...	...
10,000	.10	1,300	.2	2,034	.3	2,604	.4	3,240	...	...
15,000	.05	1,192	.10	1,584	.15	1,955	.2	2,242	.50	4,105
20,000	.025	1,264	.05	1,544	.075	1,760	.10	1,956	.250	2,958
25,000	...	...	.035	1,815	.05	1,946	.075	2,220	.175	3,050
30,000	...	...	.025	2,122	.035	2,245	.05	2,414	.125	3,140

As might be expected, for each distance stated a minimum in first cost is obtainable by varying the pressure of transmission in accordance therewith. It is interesting to note that a transmission pressure of 20,000 volts would appear most economical under the conditions assumed for distances between 10 and 50 miles.

In addition to the economies to be effected in first cost under Board of Trade Regulations, it is only right to consider the possible economies to be effected in working where a number of cables in parallel are employed to transmit a given load.

We have already seen that the copper, lead sheath, and armouring or iron trough losses depend upon the square of the load current whilst the loss in the dielectric is practically independent of the load current, unless unduly heated thereby. It is, therefore, obvious that for any number n of similar feeders in parallel:—

The copper, lead sheath, and iron losses $\propto \frac{1}{n}$, whilst the dielectric loss $\propto n$.

If we suppose the copper, lead sheath, and iron loss for the transmission of a certain number of kilowatts $= \frac{K}{n}$, and the dielectric loss with n feeders in parallel $= kn$, we have to determine the conditions under which the total loss W due to copper,

lead sheath, iron and dielectric is a minimum, that is, $\frac{K}{n}+kn$ must be a minimum. This is obviously the case when $n=\sqrt{\frac{K}{k}}$.

Take the case of 6,000 kw. transmitted 6 miles at 10,000 volts. This would mean at least six cables on the basis of 1,000 kw. per cable without taking into consideration a suitable number of spares, which would in most cases be necessary.

The full load current in the cores of each cable would be about 60 amperes.

Assume—

(*a*) Copper loss at full load, 2 per cent.	=	20 kw.
Sheath loss, 1 kw. per mile	=	6 „
Iron loss, 0.5 kw. per mile	=	3 „
		29 kw.
(*b*) Dielectric loss, 2 kw. per mile	=	12 kw.

The most economical number (n) of feeders to use in parallel under the above conditions is given by

$$n=\sqrt{\frac{29\times 6}{12}}=3.8 \text{ or } 4 \text{ nearly.}$$

If we denote by C the total load current

$$C^2\times J=29\times 6=174 \text{ kw.}$$

where J is a constant and all six cables are working at full load.

$$\therefore\quad J=\frac{174\times 10^3}{(60\times 6)^2}$$
$$=1.34.$$

For maximum economy

$$n=\sqrt{\frac{C^2\times 1.34}{12\times 10^3}}$$
$$\therefore\ n=.01055\ C.$$

We, therefore, should vary the number of feeders in parallel according to the load for maximum economy in working, as follows :—

TABLE XIII.

Amperes.	Feeders.	Amperes.	Feeders.
95	1	378	4
190	2	475	5
283	3	570	6

To summarise the preceding considerations, it would appear that maximum economy would be effected by switching off trunk mains at times of light load, varying the number in parallel according to the load curve. In connection with this point it may be observed that the oscillograph has conclusively proved that with oil switches the operation of switching off high-pressure cables is perfectly safe since the current is always broken when passing through or about the zero value. A danger exists, however, in switching on an open ended cable resulting in the formation of oscillations of double pressure, due to reflected waves from the open end. This difficulty, great as it may seem, is not by any means insurmountable. One safety method is to connect a three-phase transformer to the open end of the cable before making it live, the secondary of the transformer being closed through a water resistance, subsequently disconnecting the transformer when the cable has been switched on. A further method is to arrange the switchboard with ring bus bars at the generating station divided into sections consisting of feeder and generator panels with interconnecting switches. This allows of any one or more trunk feeders being made live gradually, and paralleled with other live feeders. The objection to this method is the cost of starting up large generators solely for the purpose of making cables live. A third method is to employ a water resistance charging gear. A still further method is the use of a motor generator to make the cable live gradually but at constant frequency. With a well-constructed paper-insulated cable, however, capable of withstanding with safety a temporary rise in pressure of three or more times the working pressure, such devices would appear to be unnecessary, and the direct switching on of such cables becomes permissible.

Kelvin's Law.—Kelvin's law states that the maximum of economy is attained in transmitting a given amount of power at fixed voltage at the receiving end of the line, when the annual cost of the C^2R loss in the line is equal to the annual interest and depreciation charges on that part of the line the cost of which varies as the sectional area of the conductors.

If the cost of one conductor of the line in £ per mile is expressed by $A+Ba$, a being its area in square inch and A and B constants, e equals rate per cent. for interest and depreciation on capital expenditure; C equals average effective current in

amperes per wire; K equals cost of generating 1 E.H.P. per annum, including all annual charges, then it may be shown that the most economical sectional area to adopt is given by:—

$$a = .0755\ C\sqrt{\frac{K}{e\,B}} \text{ for copper conductors,}$$

and the most economical current density—

$$\frac{C}{a} = 13.2\sqrt{\frac{e\,B}{K}} \text{ for copper conductors.}$$

The average cost of 1 E.H.P. per annum of 8,760 hours is approximately £6.8, with a number of hydro-electric plants, although this varies considerably with different undertakings. This figure corresponds to a total cost per unit generated of 0.25 penny.

Interest and depreciation on the capital cost of the conductors of the overhead line may be taken at 15 per cent.

The cost of the conductors will vary with their sectional area and the cost per ton of the metal of which they are composed. With stranded copper conductors, we may assume that the weight per ton per mile is:—$9.36 \times a$ approximately; where a is the total sectional area in square inch.

Thus with drawn copper at £65 per ton, the cost of 1 mile of conductor of sectional area a square inch would be £$609 \times a$.

Under the above conditions we get for our sectional area:—

$$a = .0755\sqrt{\frac{6.8}{15 \times 609}}\ C,$$

$$\text{or } a = C \times .00197.$$

Thus if C equals 100 amperes, a equals .197 sq. in., or each conductor would have a sectional area of approximately .2 sq. in. The current density is thus 500 amperes per square inch, and with this current density the drop in volts along each conductor would be about 22 volts per mile.

It is to be specially noted, however, that the above calculation takes no account of the cost of insulating the line, and a more useful condition to apply is that the cost of the C^2R losses shall be equal to the whole of the charges for interest and depreciation on the transmission line.

The cost of flexible steel supports and insulators for a line insulated for 60,000-80,000 volts may be taken at £250 per

mile. With cost of copper at £65 per ton, the cost of the conductors for a line of three wires will be $3 \times 609 \times a = £1{,}827\ a$ per mile.

Assuming a line loss of 10 per cent. and amount of power transmitted 10,000 kw. a distance of 150 miles. The wasted energy will be 1,000 kw., or $\frac{1{,}000}{150} = 6.65$ kw. per mile, and the annual value of this at 0.25d. per unit is £60.7 per annum.

This corresponds to a total capital cost for the line of £405 per mile, allowing interest and depreciation at 15 per cent.

The cost of insulation and supports is £250 per mile, leaving a balance for the conductors of £155 per mile.

If the conductors are of copper, we have, therefore,

$$£155 = £1{,}827 \times a, \text{ or } a = \frac{155}{1827} = .085 \text{ sq. in.}$$

The line loss of 6.65 kw. per mile corresponds to a loss of 2.22 kw. per wire and the resistance of a wire of sectional area a is per mile approximately :—

$$\frac{.042}{a}, \text{ or in this case } \frac{.042}{.085} = .495 \text{ ohm.}$$

Our C^2R loss, therefore, gives us :—

$$C = \sqrt{\frac{2220}{.495}}, \text{ or } 67 \text{ amperes.}$$

Now the voltage V necessary to transmit 10,000 kw. at 67 amperes per conductor is :—

$$V = \frac{10000 \times 1000}{\sqrt{3} \times 67}, \text{ or } 80{,}600 \text{ volts.}$$

The CR drop per line wire is $67 \times .495 = 33.3$ volts per mile, or for 150 miles 8,660 volts between wires, and our percentage line drop is :—

$$\frac{8660}{80600} = 10.75 \text{ per cent.}$$

The above example illustrates the necessity for high voltages being employed with power transmission over great distances in order to secure the conditions of maximum economy.

It may be of interest to consider at this point the most economical sizes of conductors according to Kelvin's law with extra high-pressure transmission cables.

If we plot on squared paper the total cost of laying similar

cables insulated for the same working pressure with the same reinstatement but having different sectional areas, we find that the corresponding values of total cost and sectional area give us points lying very nearly on a straight line.

The total cost K per mile in £ and the sectional area S are in fact for normal sections connected by the law

$$K = AS + B, \text{ where A and B are constants.}$$

Taking the cost of paper-insulated, lead-covered and armoured cables laid and jointed with those prices of copper and lead previously assumed, we find that the sectional areas and cost per mile are related to one another approximately as follows :—

Working Pressure.	Cost in £ per Mile.
30,000 - - - -	$K = 10700\, S + 1860$
20,000 - - - -	$K = 7171\, S + 1200$
10,000 - - - -	$K = 6700\, S + 650$

If we denote by p the rate per cent. required to cover interest and depreciation charges upon the cost of the cables laid, that part of these charges per annum per mile of cable which is proportional to the sectional area of the conductor is ApS, the constant A being given in the above table for different working pressures.

The cost of the wasted energy must be considered as involving extra capital expenditure on plant and buildings entailed by extra plant capacity required to supply this loss, and the interest and depreciation charges upon such capital must be included in the cost per unit of wasted energy.

If we assume a capital cost for plant and buildings of £35 per kilowatt and average interest and depreciation charges at 10 per cent, the annual cost under this heading per unit per annum is £3. 10s.

The cost per unit of wasted energy must also include the net running cost per unit with the particular station under consideration, or that cost strictly proportional to the number of units generated. This will depend upon the load factor, the cost of coal and other items in any particular scheme, and must be determined by careful analysis of the total works cost per unit. For the present purpose we may assume this to amount to 0.35d. per kilowatt hour, which corresponds to an annual cost

of £12.77. Our total cost per unit per annum for wasted energy is, therefore, made up as follows:—

Capital charges - - - - - - -	£3.5
Running „ - - - - - - -	12.7
Total - - - -	£16.2

This corresponds to a total charge of £12.07 per E.H.P. per annum.

Assuming that the energy to be transmitted per single cable is limited to 1,000 kw. under the Board of Trade Regulations, and that this energy is utilised solely for town lighting with a 13 per cent. load factor, the maximum and average currents per conductor for various transmission pressures are given in Table XIV., and we may apply Kelvin's law to ascertain the sectional areas which will be most economical under the conditions assumed. Thus, if we allow 10 per cent. for interest and depreciation charges upon that part of the cable proportional to its sectional area, we have for the 30,000 volt cable:—

$$S = .0755\sqrt{\frac{12.07}{10700 \times 10}} \times 6 = .0048 \text{ sq. in.}$$

TABLE XIV.

Transmission Pressure.	Maximum Current per Conductor.	Average Current per Conductor.	Most Economical Section of Conductor.
Volts.	Amperes.	Amperes.	Square Inch.
30,000	19.2	6	.0048
20,000	28.9	9.2	.009
10,000	57.7	18	.018

It is interesting to note the following points in connection with the above table:—

1. The most economical average current density is approximately 1,000 amperes per square inch.

2. The sectional areas are too small to be generally adopted in practice on account of mechanical and other considerations.

3. The current density at the time of maximum load, which would exceed 3,200 amperes per square inch with the load curve assumed, would be likely to cause damage to the cables from excessive heating.

4. The drop in pressure per mile of conductor at maximum load would be approximately 141 volts, or 244 volts between wires.

5. For a 10 per cent. drop in the line, the maximum distance under the most economical transmission conditions would be as follows :—

Working Pressure in Volts.	Distance in Miles.
30,000	7.1
20,000	4.7
10,000	2.3

It will thus be seen that the application of Kelvin's law, combined with the limit of 1,000 kw. per cable, leads to results which are not commercially practical under the load conditions assumed.

CHAPTER III

WORKING PRESSURE

In determining the most suitable pressure to adopt in any particular case we must take into consideration the following items:—

1. The distance to be covered by the transmission of energy.
2. The amount of energy to be transmitted.
3. The loss to be allowed in the line as governed by facilities for regulation and the maintaining of constant pressure at the receiving end.
4. The most economical size of conductor to employ, both as regards first cost and working expenses.

In connection with the above items, it is first to be noted that if we take full advantage of the current carrying capacity of any particular size of cable as limited only by the heating effect, the C^2R loss in the cable will be in direct proportion to its resistance and length, and this loss will manifest itself by a drop in pressure at the receiving end. Now although we cannot actually alter the amount of this loss with a given current and section of conductor, we can make it as small a percentage of the power transmitted as we please by increasing the pressure of transmission. Thus in the case of a three-core three-phase cable, if—

E = pressure between cores at receiving end in volts,
C = current per core in amperes,
R = resistance per core in ohms,
e = drop between cores in volts,

we have

Power transmitted $W = EC\sqrt{3}$ - - - - - (1)
Power lost in line $w = C^2R \times 3$ - - - - - (2)
Drop in pressure on line $e = CR \times \sqrt{3}$ - - - - - (3)

It is obvious that the ratio of the power lost in the line to the power transmitted expressed as a percentage loss is:—

$$\frac{w \times 100}{W} = \frac{C^2R \times 3 \times 100}{EC\sqrt{3}} = \frac{e}{E}100 \qquad (4)$$

Similarly the efficiency of transmission is :—

$$\frac{(E-e)C}{EC}=1-\frac{e}{E}.$$

It will be evident from the above that the pressure may be increased indefinitely with a corresponding decrease in the line loss expressed as a percentage of the power transmitted.

We have already seen, from a discussion of the line loss in the previous chapter that in practice the regulation to be effected at the generator end of the line to meet the percentage drop in the line itself, the transformers at the receiving end, and the distribution system is generally strictly limited, and, therefore, it becomes necessary to choose a transmission pressure which will bring the loss in pressure in the line, transformers, and distribution system within these limits of regulation. Moreover, it will be at once apparent that having fixed the total line loss or percentage drop in pressure to be allowed, this quantity expressed per mile of cable over which the transmission is effected must correspondingly decrease as the total distance is increased.

Taking the general case met with in practice we usually have

Amount of energy to be transmitted fixed,
Distance fixed,
Line loss limited by regulation,

whereas our variables are

Current density and
Pressure of transmission.

For the purpose of rapid calculation of copper losses on transmission cables it is convenient to remember the following approximate relations :—

(*a*) The resistance per statute mile of single conductor is given with close approximation by dividing the constant .0424 by the sectional area of the core in square inches.

(*b*) With a current density of 1,000 amperes per square inch the drop in pressure will be approximately 1 volt for every $41\frac{1}{2}$ yards of single core.

From equations (1) and (2) above we may readily deduce: the following approximate relations :—

$$E=\frac{\frac{7.4\times\text{Amperes}}{\text{Square inch}}\times\text{Distance in miles}}{\text{Drop in pressure per cent.}} \quad - \quad - \quad (5)$$

Similarly, if we denote the power transmitted in kilowatts by K with power factor P and percentage line loss n we deduce:—

$$E = \sqrt{\frac{4240 \times K \times \text{Distance in miles}}{A \times n \times P}} \quad - \quad - \quad (6)$$

$$A = \frac{4240 \times K \times \text{Distance in miles}}{n \, . \, E^2 \times P} \quad - \quad - \quad - \quad (7)$$

If we require to transmit the maximum amount of energy at a minimum cost under the working conditions stated, we have seen that in practice the problem usually reduces itself to the determination of the relative values of current density in the cable and pressure of transmission. The greater we make the current density the greater we must make the transmission pressure to keep the percentage line loss within the regulation limit; we shall, therefore, require to balance saving in cost of copper against extra cost of insulation entailed by the higher pressures adopted.

If we fix the power to be transmitted by the cable, we must, therefore, vary the sectional area of the cable and the transmission pressure to give us the minimum of first cost with the given line loss; as will be seen from equation (6) where E and A would be the only variables under the conditions assumed.

The smaller the sectional area the less will be the cost of copper, but the greater will be the pressure of transmission and cost of insulation.

The curves, Fig. 7, based upon the transmission of 1,000 kw. per cable, in accordance with the Board of Trade requirements, at various pressures, and a line loss of 2 per cent., will illustrate this point. It will be seen that a minimum of first cost is obtainable by suitably choosing the working pressure for any particular distance of transmission.

It is obvious, however, that we must consider the working costs as well as initial cost, to enable us to finally select the most economical cable in practice.

The application of Kelvin's law for this purpose in its ordinary form is not sufficient, since we have, in addition to a copper loss depending upon the sectional area of the conductor and current flowing, a dielectric loss, varying as the square of the pressure of transmission, and in addition proportional to the frequency.

We see, however, that definite pressure limits exist in any

particular case, first, from consideration of initial capital cost either for copper or insulation, secondly, from consideration of working cost in connection with the losses constantly going on in the cable whilst energised both in the dielectric and in the copper due to the charging current, even on open circuit.

The effect of working pressure on the first cost of the cable is illustrated by the curves given in Fig. 7. In any particular case, however, due consideration would have to be given also to the increased cost of generating plant and

Cost of Cables Transmitting 1,000 K.W. with 2 % Line Loss at Various Transmission Pressures.

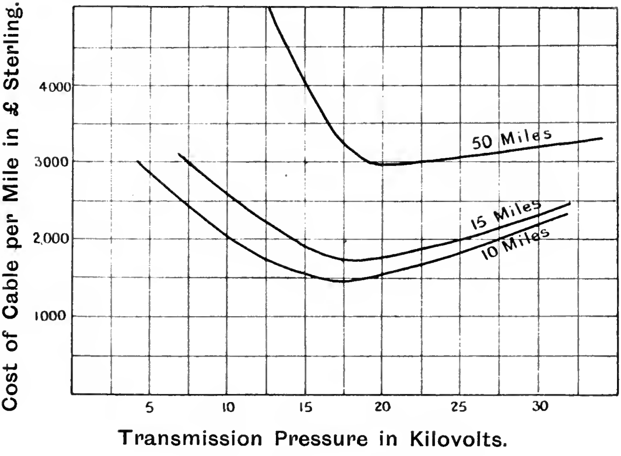


FIG. 7.

switchgear as the line pressure was raised, and in addition to the nature of the load and safety in working operations.

As regards working cost, it will be necessary to consider in somewhat closer detail the order of the losses to be expected in practice.

At the outset we are met with a divergency of opinion amongst experimenters regarding the nature of the loss in the dielectric, some asserting that this loss is due to molecular friction as in the case of magnetic hysteresis, others asserting

Temperature Test of 20,000 Volt Three-Core 7/·095 Cable.

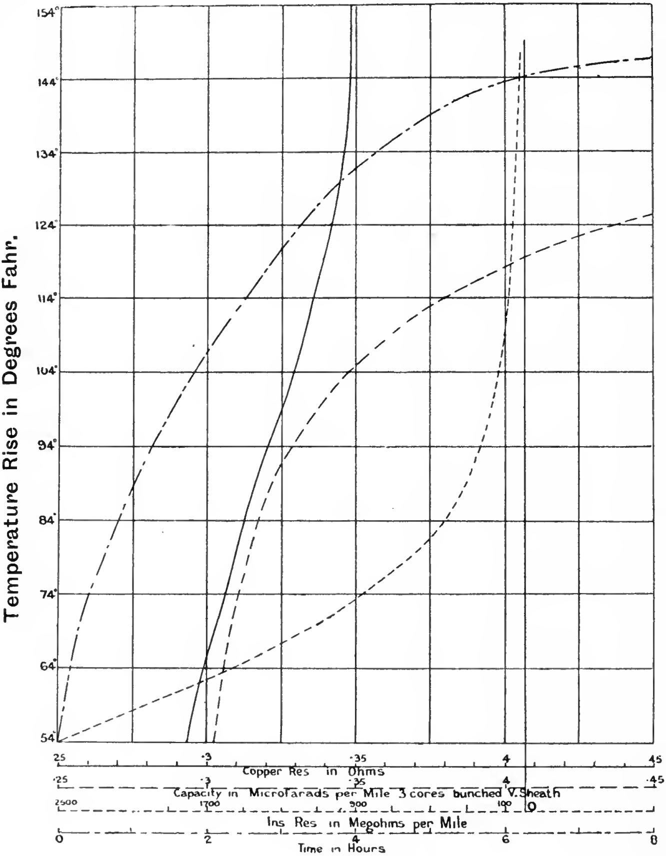


Fig. 8.

that it is due to the resistance of the dielectric as a conductor. The scope of the present work will not permit of a review of the arguments put forward by both sides. The consensus of

opinion amongst those who have experimentally studied this question would, however, appear to be as follows :—

(1) With dielectrics of impregnated paper and with the range of pressures at present employed with these cables, the dielectric loss varies closely as the square of the effective pressure, and directly as the frequency.

(2) The dielectric loss is sensitive to and varies nearly directly with the capacity, and inversely as the resistance of the insulator, when diminished by increase in temperature.

In connection with the above the curves illustrated by Fig. 8 and relating to a 20,000 volt cable may be interesting.

Upon considering the capacities of a number of cables of the same section of conductor, it will be found that there is little variation in the capacity as compared with the working pressures for which they are constructed. For instance, in the case of a three-core .04 square inch cable constructed for 5,000 volts working pressure, and a similar size of cable constructed for 20,000 volts working pressure by the same maker, the capacity of one core to two others bunched to lead sheath in the latter cable was found to be 66 per cent. of the former, and of all three cores bunched to lead sheath 71 per cent. of the former.

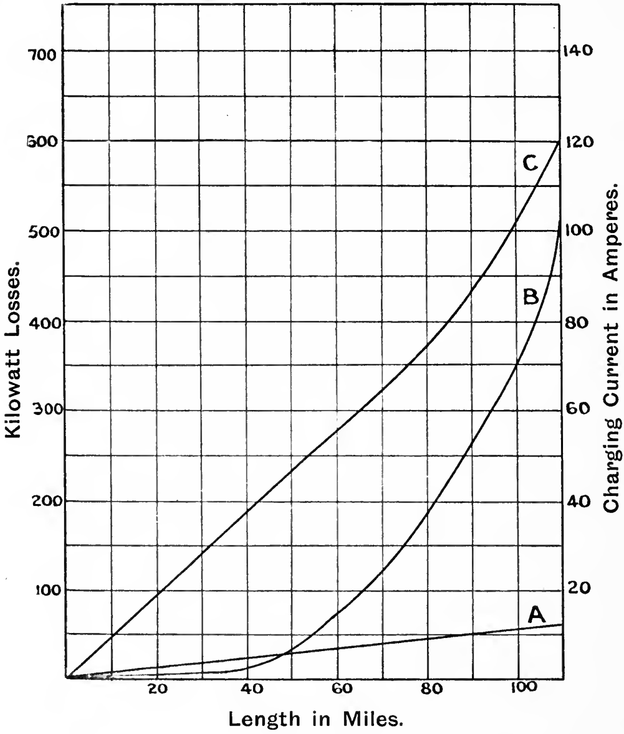
As the loss in the dielectric is found to vary as the capacity and the square of the working pressure, we may take it that for all practical purposes (in view of the variation to be found in similar dielectrics) the loss in a transmission scheme with a given size and length of cable will vary as the square of the working pressure we adopt.

The charging current may also be considered as varying directly as the working pressure. Take the case of a .15 sq. in. 11,000 volt cable, the losses with an approximate sine pressure wave are illustrated by the curves, Fig. 9. We see that with a 30-mile length of this cable the total open circuit loss is about 10 kw., being made up of 6 kw. in the dielectric and 4 kw. in the copper. At 20,000 volts these losses would be approximately 22 kw. in the dielectric and 13 kw. in the copper, or 35 kw. At a length of little over 40 miles the copper loss exceeds the dielectric loss and increases to an enormous extent with long cables.

It was stated in Chapter I. that with lighting load curves having an average load factor of 13 per cent. for summer and winter, the root mean square value of the load current through-

out the year was found to be closely one-third $\frac{1}{3.22}$ of the maximum current in the same interval. We can, accordingly,

11,000 Volt 0·15 sq. in. Three-Core Paper Cable.



A = Dielectric Loss. B = Copper Loss. C = Charging Current.

FIG. 9.

express the annual transmission loss directly as a function of the drop allowed in the line at full load with such load curves.

Upon the assumption that 1,000 kw. is the maximum load to be transmitted by any one cable, it is instructive to examine

the losses in copper and dielectric on transmission schemes over various distances under these conditions. Tables XV. and XVI. refer to 5,000 volt and 20,000 volt cables respectively. In the case of Table XV. the observed value of the charging current over a number of miles of this cable has been given. It is nearly twice the value which would have been obtained with a true sine wave.

In connection with the annual loss in the dielectric given as a percentage of the units transmitted, it is of great importance to note that if the maximum load had been less than 1,000 kw., say 200 kw., the percentage losses would be increased by five times.

It is, therefore, evident that to secure maximum economy, considerations which should govern the working pressure to be adopted should take account of:—

(*a*) Initial cost of cable, generators, and switchgear.

(*b*) Open circuit losses in the copper and dielectric of the cable.

(*c*) Load factor of the demand at the receiving end.

The scope of the present work will not permit of the following up here of this question further. Suffice it to say, however, that the transmission of electrical energy upon a remunerative basis can only be effected by due and proper regard being given to the issues indicated in the foregoing remarks.

Breakdown Strength of Dielectrics.—In connection with the subject of working pressure, it may not be out of place at this stage to refresh our memories regarding some properties of dielectrics.

Text-books on electrostatics define the unit of quantity of electricity as a charge which, when placed at a distance of one cm. in air from a similar and equal quantity, repels it with a mechanical force of one dyne. Similarly, potential is measured by the work done in moving a unit of +electricity against the mechanical forces exerted upon it, unit difference of potential existing between two points when it requires the expenditure of one erg of work to bring a unit of positive electricity from one point to the other against the forces exerted on it. Now, it will be remembered that work is defined as the product of force by the distance through which the force is overcome, and, therefore, if the difference of potential between two points is the

4

TABLE XV.—5,000 VOLT .15 SQUARE-INCH THREE-CORE CABLE.

Wave Form as per Fig. 46.

	Length of Cable in Miles.						
	5	10	15	20	25	30	35
Charging current per core = C_k amps.	4.35	8.7	13.05	17.4	21.75	26.1	30.45
Resistance in ohms per core = R	1.430	2.86	4.29	5.72	7.15	8.58	10.01
Copper loss in kw. = $C_k^2R \div 1{,}000$	.027	.216	.732	1.73	3.38	5.86	9.30
Dielectric loss in kw. = $\frac{VC_k\sqrt{3}\times .028}{1000}$	1.05	2.11	3.17	4.23	5.28	6.34	7.39
Total loss in kw., copper plus dielectric	1.077	2.33	3.90	5.96	8.66	12.20	16.69
Annual loss in dielectric as per cent. of units transmitted with 1,000 kw. maximum load at 13 per cent. load factor	.812	1.625	2.44	3.25	4.06	4.88	5.68
CR drop as per cent. of pressure of transmission at full load of 1,000 kw. P.F. = 0.8	7.18	14.35	21.5	28.7	35.9	43.1	50.2

TABLE XVI.—20,000 VOLT .05 SQUARE-INCH THREE-CORE CABLE.

Pure Sine Wave Form.

	Length of Cable in Miles.					
	5	10	20	30	40	50
Charging current per core $=C_k$ amps.	3.75	7.5	15	22.5	30	37.5
Resistance in ohms per core $=R$	4 3	8.6	17.2	25.8	34.4	43.0
Copper loss in kw. $=C_k^2R \div 1{,}000$	.06	.48	3.88	13.06	30.96	60.45
Dielectric loss in kw. $=\frac{VC_k\sqrt{3}\times .028}{1000}$	3.64	7.27	14.55	21.82	29.1	36.4
Total loss in kw., copper plus dielectric	3.70	7.75	18.43	34.98	60.06	96.85
Annual loss in dielectric as per cent. of units transmitted with 1,000 kw. maximum load at 13 per cent. load factor	2.8	5.6	11.2	16.8	22.4	28
CR drop as per cent. of pressure of transmission at full load of 1,000 kw. P.F. $=0.8$	1.34	2.68	5.36	8.04	10.7	13.4

work done in moving a +unit from one point to the other, the average electric force between the points will be found by dividing the work done by the distance between the points. That is if V*a* and V*b* are the potentials of the inner and outer conductors of a concentric cable with dielectric of thickness D, the average electric force in the dielectric is—

$$\frac{Va - Vb}{D}.$$

We see from this that as D is diminished indefinitely the force becomes nearly uniform, and the electric force at a point within the dielectric is given by the rate of change of potential at this point. Thus the resultant electric stress at a point in the dielectric is sometimes termed the electric intensity or potential gradient.

Now, the result of subjecting any material substance to stress is to produce strain or molecular displacement, and if the stress be further increased, finally rupture of the material ensues.

An important difference between the behaviour of dielectrics subject to electrical stresses and materials subject to mechanical stresses must, however, be noted at this point, and that is the property of some dielectrics to act as electrolytes or conductors under excessive electrical stresses. An interesting illustration of this is the difference to be found in the sparking distance between points and spheres subject to the same voltage. Owing to a brush discharge occurring at much lower voltage between needle points, the sparking distance between points is greater than between spheres, the explanation being that the air surrounding the points is acting as an electrolyte or conductor under the excessive electrical stress, and behaves in the same manner as a sphere surrounding the needle point, thus for all practical purposes reducing the distance between the points.

The idea given by the above illustration is of importance, and when applied to a cable with a solid dielectric we may imagine the material yielding to the pressure up to a certain point within it, thus acting as a conductor and absorbing energy. A further interesting experiment illustrating this point is the following:—

If we have two conductors separated by an air space, and an alternating difference of potential is maintained between them just below that necessary to produce disruptive discharge,

and some insulating material be then introduced between them having a greater specific capacity than air, the air and insulating material both break down. The accepted explanation is that since the potential gradient in the air in the first instance was the steepest it could withstand, and the increased specific capacity of the insulating material causes the potential gradient to be less steep within it than the air, the result is an increase in the potential gradient in the remaining air space, which first gives way and is followed by a breakdown of the insulating material.

The dielectric strength of an insulator may be defined as the greatest electric stress it can withstand. The dielectric strength of liquids and liquefiable solids, such as paraffin, wax, &c., can be readily determined by measurement of the disruptive voltage between two equal spheres embedded in the material. In the case of paper and other similar dielectrics, the measurement of the dielectric strength presents some difficulty. Sheets of insulating material placed between metal electrodes and subject to alternating electric pressures cause the air in the neighbourhood of the electrodes to be ionised, disturbing uniformity in the temperature of the dielectric and the corresponding maximum stress to which it is subjected.

To fix our ideas we may note the following results obtained by various experimenters, the dielectric strength being expressed in each case as the potential gradient in kilovolts per centimetre the material will withstand :—

Dielectric.	Dielectric Strength.
Manilla paper impregnated with resin oil	250
Paper, beeswaxed	540
Paper, paraffined	360
Resin oil	270-1,350
Vulcanised rubber	476
Gutta-percha	109
Air	27

Points of great interest in connection with the above are that insulators which heat up when subjected to alternating pressures do not heat up when subjected to continuous pressure, also that no brush discharge or hissing occurs in the neighbourhood of the breakdown stress if direct pressure be employed, in addition that the *time* the electric stress is in operation largely

affects the result. For instance, presspahn, 5 mm. in thickness, was found to be punctured in thirty seconds with 11,000 volts, and in two minutes fifteen seconds with 9,000 volts. Similarly, marble 20 mm. in thickness was punctured in seventy-eight seconds with 20,000 volts, and in two minutes by 15,000 volts.

In Chapter I. reference was made to the "Corona" effect met with on overground transmission lines. It may now be of interest to consider this point in further detail.

When bare conductors opposed to one another are subject to a very high potential difference between them, a faintly luminous glow, blue in colour, surrounds them, and at this stage a loss of power from atmospheric dispersion commences. If the potential difference between the conductors be still further raised, a brush discharge occurs accompanied by hissing and the loss of power greatly increases. It is important to note that this brush discharge takes place from the exterior of the luminous glow previously mentioned, and not from the surface of the conductors themselves. Experiments show that in the space occupied by the glow the air is partially ruptured, and is, in fact, conducting, and that electrostatic stresses in the air space between the conductors then start from the exterior of the glow. Further experiments point to the fact that a layer of air immediately surrounding a conductor, and which is found to vary in thickness with the diameter of the conductor, has a resisting power to break down many times that of the remaining air lying between the conductors. The critical voltage of any circuit, or that voltage at which the "Corona" is produced, followed by a loss of energy from atmospheric dispersion, is found to depend upon atmospheric conditions such as barometric pressure, humidity, and others which have probably not yet been investigated. The critical voltage of the circuit also becomes higher as the diameter of the wires and their distance apart are increased, but the effect of increasing the spacing of the wires upon the critical voltage quickly reaches a limit, and any further spacing is practically ineffective in preventing the formation of the "Corona." It has been found that whilst the effect of rain is small, yet the presence of fog, smoke, and particles in suspension in the atmosphere largely increase the losses. Further, these depend in every case upon the maximum value of the voltage wave, and also to some extent upon the frequency of the circuit.

Some observed atmospheric losses on different lines are given below :—

TABLE XVII.

Diameter of Wires.	Distance apart of Wires.	Effective Working Pressure.	Loss per Mile.
Inch.	Inches.	Volts.	Kilowatts.
.325	84	110,000	3
.325	52	58,000	0.89
.325	35	51,000	0.22

The "Corona" effect has been investigated by Ryan and Mershon in America, but the results obtained by these experimenters show considerable discrepancies.

Kapp gives the following formula based upon Mershon's results for the critical potential difference in virtual kilovolts between two parallel wires :—

$$\text{K.V.} = \frac{0.115\, b}{0.5 + r} \left(\frac{1}{1 + 0.013\, v}\right) r \log. \frac{s}{r}$$

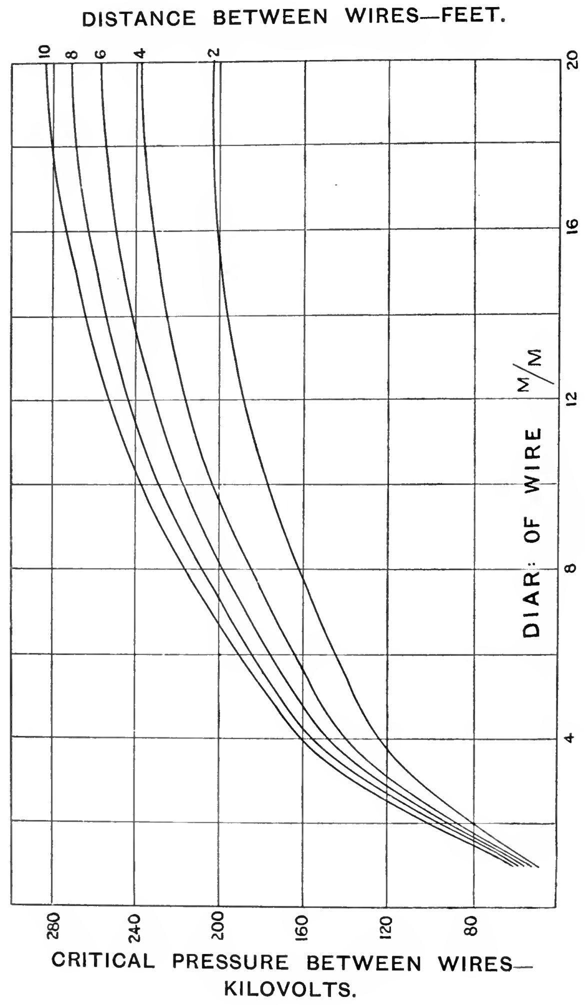
where b = barometric pressure in millimetres of mercury.
r = radius of wire, centimetres.
s = distance between wires in centimetres.
v = Mershon's vapour product or the pressure of saturated steam in millimetres of mercury at the given temperature multiplied by the relative humidity or the ratio

$$\frac{\text{actual moisture}}{\text{possible moisture}}.$$

Curves based upon this formula, giving critical pressure for wires of various diameters spaced at different distances apart, are plotted in Fig. 10.

It would appear necessary, however, to employ a factor of safety of at least two with these results where the wires pass in the neighbourhood of towns or industrial districts.

It is further to be noted that dispersion loss from the wires, depending as it does upon their spacing in air, will be likely to be increased at the points of suspension where steel towers are employed, the air space between the wires being less in the neighbourhood of the metal tower than at other points along the span.



DISTANCE BETWEEN WIRES—FEET.
10
8
6
4
2
DIAR: OF WIRE M/M
20
16
12
8
4
280
240
200
160
120
80
CRITICAL PRESSURE BETWEEN WIRES—KILOVOLTS.

FIG. 10.

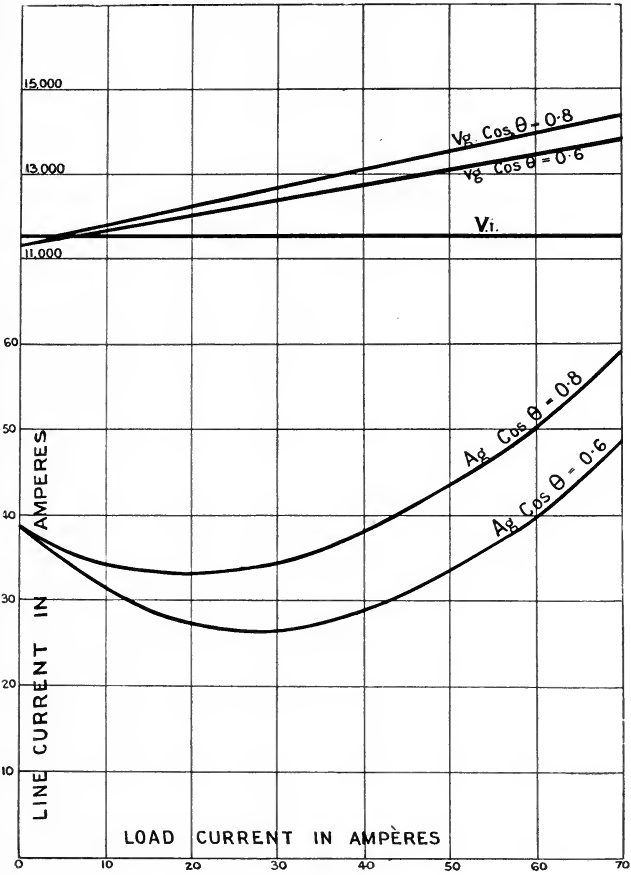

15,000
13,000
11,000
Vg. Cos θ = 0·8
Vg Cos θ = 0·6
Vi.
60
50
40
30
20
10
LINE CURRENT IN AMPERES
Ag Cos θ = 0·8
Ag Cos θ = 0·6
LOAD CURRENT IN AMPÈRES
0
10
20
30
40
50
60
70

FIG. 11.

Line Regulation.

In the western parts of America the high frequency of 60 cycles, which is common there, gives rise to very heavy capacity currents at the high working pressures adopted.

The effect of these capacity currents upon the regulation for constant pressure at the receiving end of the line is very marked in some cases. For instance, it may happen that the current entering the line at the sending end may decrease with increase of load. Further, that the generator pressure at the sending end will, under some conditions, be less than the pressure at the receiving end of the line. Some of these effects are illustrated by the curves given in Fig. 11.

In the figure the voltage V_g between each phase and neutral point of the generator required to maintain the constant voltage V_t at the receiving end of a feeder 50 miles in length, with different loads and power factors, is given by the upper curves. The current A_g supplied to the line by the generator under the conditions of different loads and power factors is given by the lower curves.

A simple and rapid method which may be adopted for estimating such effects with any given transmission scheme will be found in Appendix D.

CHAPTER IV

THE CONTROL OF E.H.P. TRUNK MAINS

Switchboard Construction.—With the rapid growth of electricity supply systems and the enormous outputs of modern power stations, the necessity for absolute continuity in the supply has become of the greatest importance. The possibility of a complete shut down of the whole supply as the result of a single fault upon a main switchboard such as the failure of an oil switch, a temporary short circuit to frame or between conductors, can, in view of such requirements, no longer be permitted. The complete destruction by fire in some cases of congested types of switchboards comprising generator, feeder and section panels all crowded into the minimum possible space, and formerly so general, has demonstrated the necessity for very wide subdivision of the controlling switchgear where large amounts of power have to be dealt with. Accordingly, the generator, feeder, and section panels comprising a modern switchboard are usually so widely separated individually and collectively, that the spread of fire is effectively limited to the faulty section, and thus disorganisation of the supply, in the event of a fault, reduced to a minimum.

Such wide subdivision involves the use of remote control switchgear in order that distant switches may be promptly operated from a keyboard situated at some convenient central point. The remote control systems in most general use are:—(1) Electrically operated; (2) Mechanically operated.

In America electrical control is largely adopted, and this system has also been installed with important plants in this country. Mechanical control has, however, been extensively used upon the Continent with remote switchgear.

On account of the considerable weight of the high-power oil switches required in modern generating stations and the long break necessary, trouble has sometimes been experienced in the closing of such switches with sufficient rapidity for synchronising

purposes where mechanically operated remote control has been installed. With electrical control the action of the switch may be made very prompt, and is unaffected by the distance the switch is situated from the control desk of the operator. It is to be specially noted that any feeder switch in connection with the main bus bars of the generating station may, under the conditions of a short circuit, have to break the total output of the plant running at the time quite irrespective of the normal load the feeder will carry. The advantage of having such switches installed with plenty of space and at a distance from the operator is obvious, and it is now quite common with high-power generating stations to find a complete side or end of the building partitioned off for the installation of switchgear, often occupying three or more floors. The arrangement of switchgear and transformers illustrated by Fig. 12 was installed by the Oerlikon Company in a Continental generating station feeding five overground transmission lines at a pressure of 33,000 volts between wires. Bare copper conductors from each 6,000-volt three-phase generator enter the switchgear annexe at the point A, and are brought to electrically operated oil switches B, fitted with maximum reverse relays. From this point connections are made through current transformers to change-over oil switches E, permitting of each generator being coupled to the 6,000-volt bus bars shown, or alternatively to the primaries of one group of three delta-connected transformers G. The secondary connections to these transformers are brought through oil switches J, fitted with maximum relays, to 33,000-volt ring bus bars at M. From these bus bars each set of feeder connections passes through electrically operated oil switches O and choke coils Q to the transmission lines leaving the building at R. Switches for isolating the lines are shown at S and lightning arresters at T in series with water resistances U.

Situated upon a platform overlooking the engine-room is a main switchboard W, controlling the transformer and line switches, and at V are pillars for operating the generator switches whilst facing the engine-room.

As will be seen from Fig. 12, the cellular system of switchgear has been adopted, the switchboards being constructed of incombustible material with horizontal and vertical partitions separating conductors. No additional precautions, however, are taken to screen live parts, but the rooms in which these are

situated are locked off from the rest of the station, and access to them on the part of all but skilled assistants is forbidden. It will further be noticed that no high-pressure connections are brought on to the operating platform.

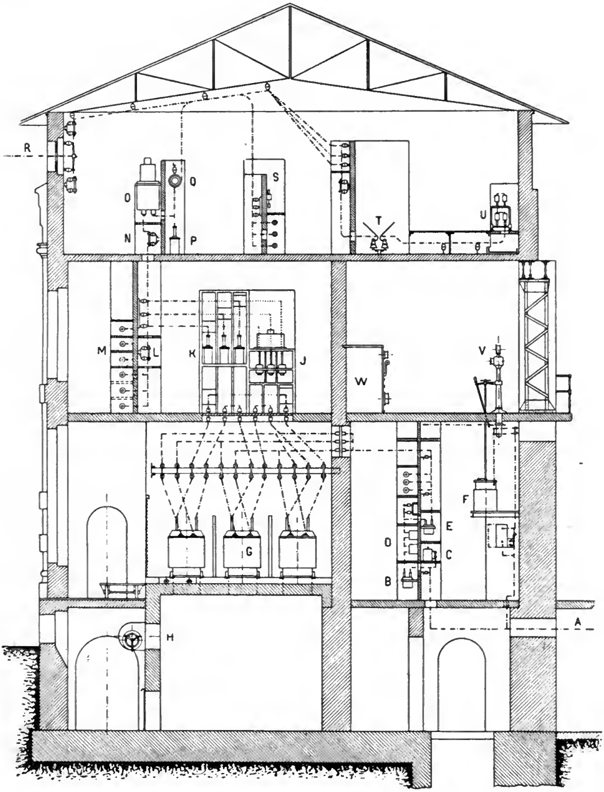


FIG. 12.

The arrangement of a bus bar room with Continental type switchgear is illustrated by Fig. 13.

As a typical arrangement of high-power British switchgear

we may take an installation by Messrs Ferranti where the feeders consist of underground cables and the following disposition of the gear is adopted. The generator and feeder panels are placed

Fig. 13.

upon opposite sides of a brick wall passing upwards through three or more floors of the building. On the first floor are situated the cable receivers, from which point the connections

of each phase pass upwards, separated by incombustible partitions forming with horizontal partitions tiers of cells containing the isolating switches, spark gaps, current and potential trans-

FIG. 14.

formers. The connections then pass through porcelain insulators to the second floor, upon which are placed the main electrically controlled oil switches. After traversing these switches the connections are brought upwards to a further

set of isolating switches, enabling each generator or feeder to be coupled to either of two sets of bus bars arranged in ring form and situated on the third floor.

Fig. 14 illustrates the arrangement of a main switch floor, and shows the electrically operated switches disposed in brick cells with the isolating switches above them.

Each floor containing extra high-pressure switchgear is locked off by doors from the remainder of the building, but everything is readily accessible to authorised persons entering the enclosures, upon the removal of light sheet-iron covers screening the cubicles and bus bar chambers. The whole of the electrically operated switchgear is controlled from a working platform overlooking the engine-room, upon which everything is safe to handle.

In the case of switchgear situated in substations at the receiving ends of lines or feeders, the working conditions are somewhat different to those at the generating station. Space will often be restricted, demanding more or less concentration of the switchgear; in addition, greater precautions will generally be necessary to protect the substation attendants from accidental contact with live parts. Since the effect of short circuits will not be so disastrous at the receiving ends of long feeders adequately protected at the generating station, it will generally be permissible to adopt self-contained designs of switchgear. At the same time, well-recognised principles must be borne in mind, and a brief review of some types of substation switchboards at present in use may not be out of place at this stage.

These may be classed as follows:—

(1) Enclosed cell or cubicle type, frame of brick, slate, or concrete, without space at the back.

(2) Flat type with space at the back, framework of metal, screening live fittings, or separate locked chambers enclosing exposed live fittings.

(3) Ironclad or solid type, frame of iron or other metal enclosing all live parts, the space between being filled up solid with insulating compound or material.

The conditions to be aimed at in the construction of a substation switchboard are, in relative order of importance, it is suggested, as follows:—

(*a*) The absence of danger to the life of the operator.

(*b*) Freedom from breakdown and effective restriction to the spread of fire.

(*c*) Reasonable initial cost.

Considering first the most important object to be attained, viz., safety to human life, it may be noted that the possibilities of the operator obtaining fatal shocks are (1) the forming of a path through his body or portion thereof for an E.H.P. discharge between live metal at different potentials; (2) the forming of a path through his body or portions thereof for a discharge from live metal to metal or other material at earth potential.

In general a discharge through the body of the operator to earth would be effectually guarded against if the material used in the construction of the switchboard frame was possessed of sufficiently insulating qualities added to the adoption of rubber mats upon the operating platform. A difficulty arises, however, from the fact that slate, concrete, brickwork, and other materials commercially adaptable for switchboard construction are not only in themselves possessed of insufficiently insulating qualities to prove a safeguard to life in cases of leakage, such as that due to a broken insulator permitting live metal at extra high pressure to come in contact with or discharge to them, but in addition such materials will of themselves pass a sufficient current at extra high pressures to fire inflammable insulating materials in their vicinity. From this it is sometimes argued that imperfect insulating material used to screen live metal is worse than bare live metal itself in at least that there is no *guise* of safety.

Accordingly, a frame of iron or other metal effectively earthed is now frequently employed to screen extra high-pressure switchboard panels, and bare copper conductors supported upon porcelain insulators are used in place of rubber or tape insulated connections.

The adoption of this course completely shields the operator from the danger of high resistance leaks and the charging of isolated metal parts thereby. It ensures also that the development of a fault will be promptly followed by a direct discharge to the earthed framework of the switchboard.

In the absence of adequate subdivision of the feeder panels, however, a discharge to frame would be likely to result in a shut down of the supply, and unless each panel be so arranged that access to the connections shielded by the metal frame cannot be gained whilst the panel is live, an increased risk of accidental

contact on the part of the operator between live metal parts and the earthed frame of the switchboard will exist.

In connection with this point must be mentioned a novel departure from the general lines upon which switchboards have for many years been constructed. This refers to the ironclad type of switchgear introduced by Messrs Reyrolle & Co. In this type the bus bars are enclosed in a longitudinal cast-iron case filled in solid with compound. Each feeder cable is

FIG. 15.

terminated by a cast-iron dividing box to which is attached directly a further casing containing current transformers and situated immediately below the bus bar chamber.

Plug sockets are arranged with suitable centres in the bus bar chamber and transformer casing respectively.

An oil switch in the form of a carriage and provided with external contacts is supported by a framework and plugs into the sockets formed in the bus bar chamber and transformer casing, thus completing the circuit between the feeder cable and

bus bars. This type of switchgear is illustrated by Figs. 15 and 16.

By this arrangement all concrete and brickwork are

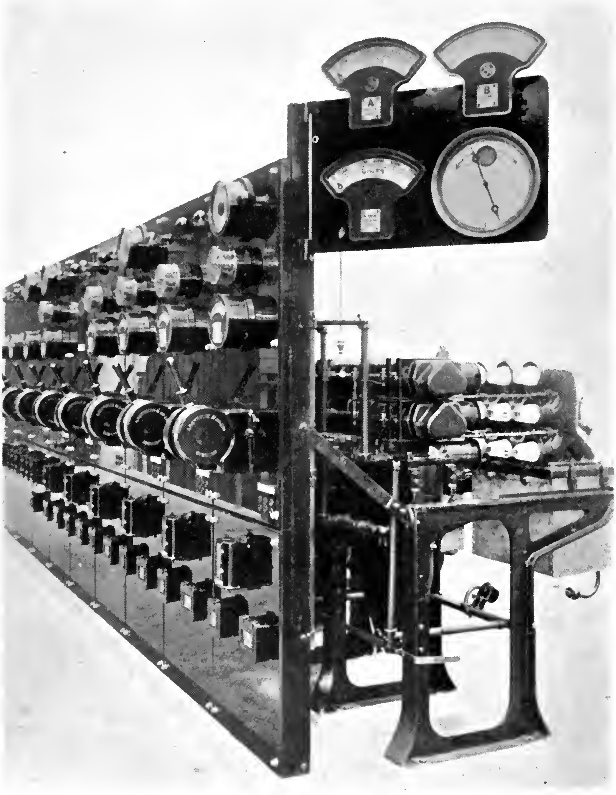


FIG. 16.

eliminated, isolating switches are unnecessary, and instrument connections rendered as short as possible. Other important features are that the amount of cleaning of live parts, insulators,

&c., is reduced to a minimum. Spare switches can be carried, and duplicate bus bar chambers readily arranged if desired.

Coming now to condition (*b*), *i.e.*, freedom from breakdown and restriction to the spread of fire if started, one of the most important points in the writer's experience is the avoidance of rubber insulated cable connections upon the switchboard. Rubber under the influence of ozone, always present in more or less quantities in the neighbourhood of extra high-pressure switchgear, is rapidly perished and rendered useless both

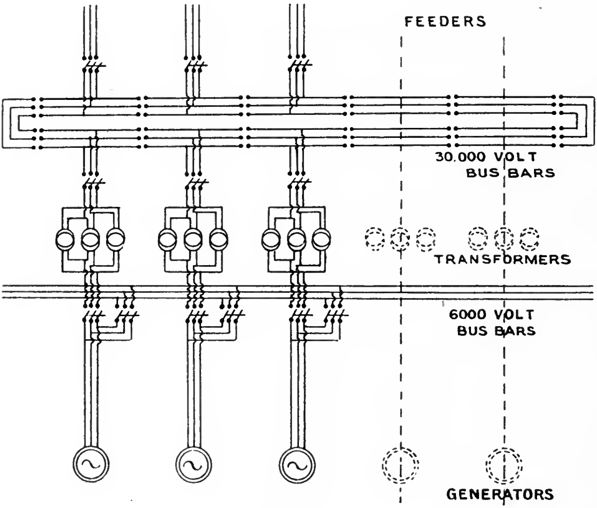


FIG. 17.

electrically and mechanically, and will readily start a fire under such conditions.

The general subdivision of phases starting from the cable receiver itself is highly desirable, but will add considerably to the cost of the switchgear, and with a brick or concrete framework generally involves deep and narrow cells, which render access to connections for repairs and cleaning difficult.

The cubicle system carried out completely in brickwork or artificial stone, however, is capable of withstanding for some

time the high temperature of heavy current arcs under the conditions of short circuit, which may allow of a fault clearing itself if the oil switches are capable, as they should be, of breaking the short circuit current.

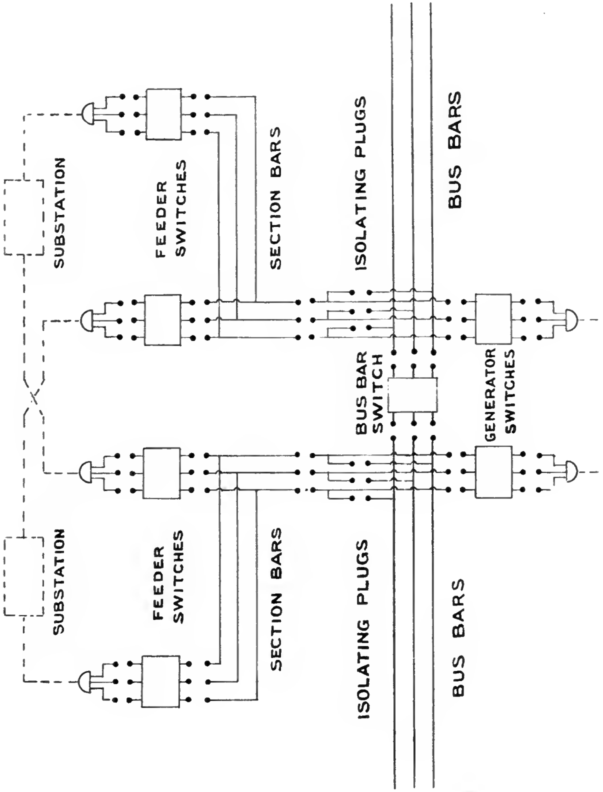


Fig. 18.

With duplicate feeders to each substation fed from independent switchboard sections an extensive interruption within the supply area is rendered a very remote contingency.

As regards condition (*c*), *i.e.*, that of reasonable cost, it is to

be noted that the use of an iron frame will largely cheapen the design of a switchboard. It must be remembered, however, that flat type switchboards with live connections exposed at the back were at one time largely used in this country, but were subsequently abandoned on the score of danger to the operator. It is very difficult to estimate the ultimate pecuniary loss accruing to an electricity supply undertaking as the result of a breakdown, but the more extensive and important the field covered by the undertaking, so much more surely will additional capital expenditure upon the score of safety and reliability be justified.

Some typical arrangements of bus bars, generator, and feeder circuits are illustrated by Figs. 17 and 18.

Where a number of trunk mains enter a distributing station from which radiate a number of subfeeders to various substations, some important considerations arise.

If trunk mains and feeders are coupled up to ring bus bars and worked undivided, a single fault upon the switchboard will usually mean a total shut down of the supply.

By allocating to certain trunk mains definite sections of the ring bus bars and their feeder circuits the effect of a fault is limited to a particular section, but the convenience of running with the trunk mains in parallel and equally loaded is lost.

A further possible arrangement is to parallel all the trunk mains on to an independent set of auxiliary bus bars to which sections of the ring bus bars with their feeders are coupled through maximum automatic switches. This allows of all trunk mains being run in parallel, and also limits the possibility of a total shut down of the supply.

It is, however, difficult even in this case to provide against all eventualities. For instance, should it happen that a fault occurred upon a feeder section of the switchboard at time of light load, the automatic switches on one or more small generators working at the time might be opened before the switch on the bus bar section upon which the fault occurred, thus causing a complete interruption in the supply.

The Merz-Price balanced system of protection, to be described later, might, with some modifications, be adapted to meet most working conditions in the sectionalising of large switchboards, and ensure that a minimum portion of the supply was interrupted in the event of a fault occurring upon any particular panel.

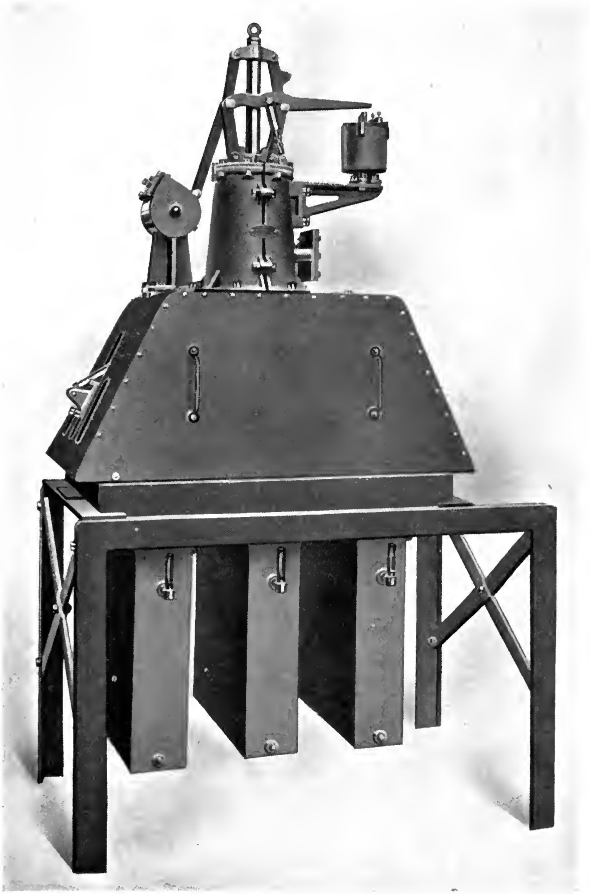


FIG. 19.

Switches.—The development of the oil type of E.H.P. switch and the satisfactory manner in which such switches are found to operate upon E.H.P. alternating current circuits

possessing self-induction and capacity, has rendered working pressures possible with safety which would hitherto have been associated with considerable risk to the cable system from surges and pressure rises attendant upon the use of air break switches. Oil break switches, although they may be relied upon to open E.H.P. alternating current circuits with safety, have no restraining influence upon the pressure rises due to the closing of the circuit. Oscillograph records show that pressure waves of about twice the normal amplitude may occur under such conditions.

Types of high-power British and Continental switches are illustrated by Figs. 19 and 20 respectively. Fig. 19 illustrates the Ferranti-Field type of electrically operated oil switch for normal working pressure of 10,000 volts and 10,000 kw. capacity. For higher voltages, three separate single-phase switches are used, each situated in its own cell, but all coupled to a single shaft operated by the solenoid surmounting the switch as in Fig. 19. By employing the direct vertical pull of a powerful solenoid to operate the switch, complication is avoided. The switch is held closed by a double scissor arrangement of links, and the tripping solenoid shown to the right of the figure allows the links to collapse and opens the switch under the action of gravity assisted by spiral springs. On the left of the figure will be seen a small barrel-type contactor for indicating by means of signal lamps whether the switch is open or closed.

Fig. 20 illustrates an Oerlikon type of mechanically-controlled switch. For pressures of 30,000 volts and upwards the three double-break switches of which it is composed are installed in separate cells. These switches are sometimes operated from a distance electro-magnetically, in other cases by means of grooved wheels and rope gear.

Before discussing in detail automatic devices including trip coils, time element relays, &c., as generally applied to E.H.P. oil switches, it may be advisable to consider some of the conditions which have to be met in practice by such apparatus.

The necessity for ensuring the continuity of the supply, and the Board of Trade Regulations restricting the total amount of energy to be transmitted by any one cable, result in the use of at least two or more cables in parallel under normal conditions in practice. As will be shown in what follows, the use of a

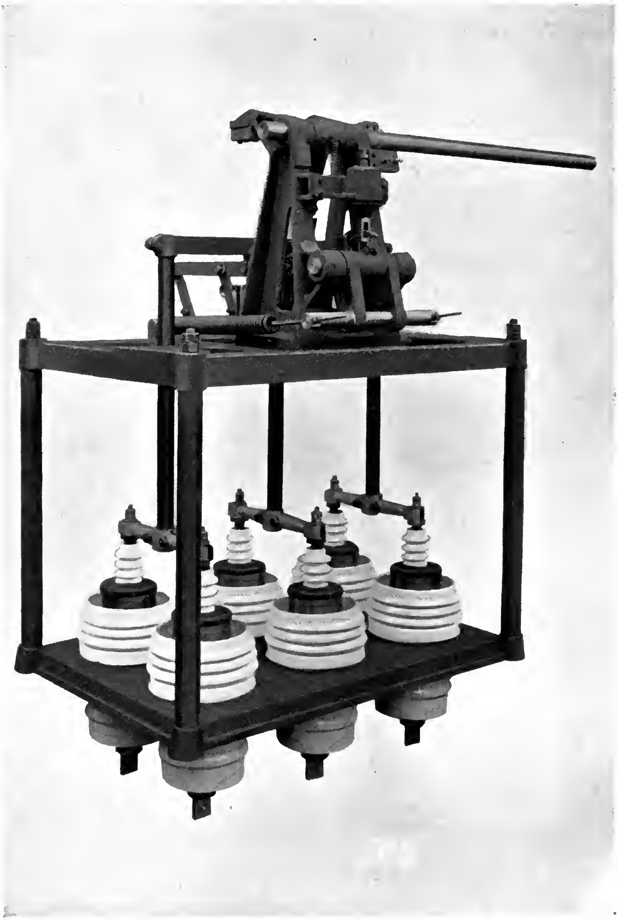


Fig. 20.

number of cables in parallel and the methods to be adopted to suitably protect them and control their working with due regard

to continuity in the supply of energy transmitted, involve some special considerations.

One point which is at once obvious is that if we have a star-wound generator with earthed neutral point supplying two feeders in parallel at the bus bar A (Fig. 21), and provided with equal fuses F_1, F_2, F_3, F_4, at the generator and receiving ends of the line A and B respectively, then if a fault occurs at E, fuse F_3 will first blow, being in the path of least resistance to the fault. This fuse will be followed by the blowing of fuses F_1 and F_2, since these, in addition to having to carry the total load being transmitted, have to carry the current through the fault by way of F_1, F_2, F_4, and E.

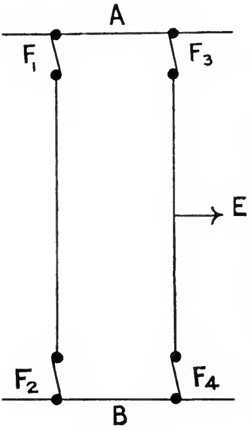


FIG. 21.

It is important to note that as we increase the number of feeders in parallel, the danger of a fault upon one feeder resulting in a total cessation of the supply may be avoided by suitably choosing the overloads at which the fuses will act.

The general case may be stated thus :—

Let Lm = Maximum load in kilowatts carried by any trunk feeder.
r = Overload required to blow each fuse in per cent. ÷ 100.
n = Number of trunk feeders.

Then the total load required to operate any fuse is :—

$$Lm(1 + r) \text{ Kw.}$$

With one feeder cut out, the load on each of the remaining $(n-1)$ feeders is increased to—

$$Lm\frac{n}{n-1} \quad . \; . \; . \quad \text{Kw.}$$

Therefore the margin before the $(n-1)$ fuses operate is :—

$$Lm(n-1)\left((1+r) - \frac{n}{n-1}\right)$$

and this must be greater than :—

$$Lm(1+r)$$

$$\therefore \quad (n-1)(1+r)-n>1+r$$

$$\text{or } r>\frac{2}{n-2}.$$

Applying the condition for safety, *i.e.*, $r>\frac{2}{n-2}$, to different numbers of feeders in parallel, we obtain values for the overload setting of the fuses as follows :—

n	r
2	∞
3	2
4	1
5	0.6
6	0.5
8	0.3
10	0.25

Thus with four feeders the overload would be 100 per cent., with six feeders 50 per cent., and with ten feeders only 25 per cent. for safety under the conditions of the fault assumed.

It will be obvious that the same law must govern the setting of simple maximum type automatic switches or cut outs. Automatic devices, however, allow of the case considered being treated in other ways.

As oil break switches became more generally used with high-pressure alternating current schemes, and the electrical advantages of the oil break became more fully understood, fuses were replaced by automatic attachments to the oil switches. It soon became evident, however, that an important feature which most types of fuses possessed in having a "time element" was in many cases still desirable with automatically operated oil switches. This led to the design of numerous devices which have now become familiar under the title of inverse time element relays or time limit relays. The control of automatic oil switches by such devices generally involves the use of three distinct pieces of apparatus.

1. Current transformer.
2. Relay with time element.
3. Trip coil.

In some cases, also, an auxiliary source of power such as a

battery is required to operate the trip coil, this being brought into circuit by the relay.

The current transformer has two principal functions. It insulates effectively the high-pressure system from the relay and trip coil mechanism, enabling the latter together with the necessary measuring instruments to be actuated at low voltage.

In addition, it enables a standard type of relay, trip coil, ammeter, &c., to be adopted upon circuits carrying currents differing very largely in magnitude. The usual practice is to wind the current transformers with a few standard ratios which give a fixed secondary current of from 8-10 amperes at the full load range of primary currents usually met with.

The function of the relay is to throw into circuit the trip coil when a predetermined overload has been reached, and after the lapse of a time interval inversely proportional to the magnitude of the overload when this is maintained.

The trip coil, as its name implies, usually actuates an armature, which by its movement trips a catch holding the oil switch in its "on" position, the switch being then opened by gravity or springs.

One of the simplest types of current transformer is constructed in the form of a ring of iron stampings upon which is wound the secondary or low voltage winding. This ring is slipped over a porcelain insulator through which a copper rod passes, carrying the high-pressure current which is made to form the primary of the transformer.

Relays.

An early form of relay used with automatic switches for introducing a time element consisted of a solenoid combined with a dashpot. The piston of the dashpot was formed of a double bell submerged in mercury and oil. If a short circuit occurred the mercury and oil contained in the lower and upper half of the piston respectively were lifted bodily, and the action of the relay was instantaneous. With an ordinary overload, however, the pull upon the plunger would regulate the time taken for the oil to pass from the upper to the lower portion of the bell through a small hole in the diaphragm, and thus allow the plunger to rise and complete the trip coil circuit.

This type of solenoid relay was found to possess many defects in practice.

After the plunger had once risen with an overload and the current then fell the plunger remained in a higher position than that originally corresponding to the current passing. If a second overload then occurred the time element was impaired if not altogether wiped out.

A further trouble sometimes arose from the fact that the impedance of the coil varied with the position of the plunger, the choking effect increasing as the plunger rose in the solenoid.

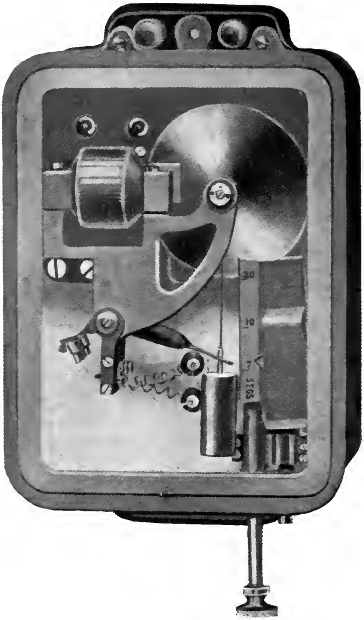


FIG. 22.

Some solenoid and plunger types of trip coil also suffer from similar defects. With a gradually increasing overload the plunger may creep up the solenoid until it comes in contact with the trip lever. A much greater overload is then required to operate the trip than if this were applied suddenly, owing to the absence of the inertia gained by the plunger by a rapid upward movement, which would otherwise ensure the knocking off of the trip. With intermittent overloads, therefore, considerable care must be exercised in the selection of a solenoid type of relay or trip coil to ensure satisfactory working.

The relay has been omitted altogether by some manufacturers, a time element being provided instead, by means of a fuse shunting the trip coil. In connection with this arrangement, however, it is to be noted that the time element of the

fuse will in general alter with its current carrying capacity, and accordingly a wide range of current transformers with various ratios of transformation become necessary to render the same type of fuse suitable for different circuits. Moreover, under the conditions of short circuits the time element of fuses disappears, and it is difficult to ensure that discrimination as to their succession of operation with two or more of such devices on the same circuit shall be retained.

As a modern type of relay we may select a well-known form illustrated by Fig. 22, made by Messrs Ferranti. This relay consists of a metal disc rotating between a pair of shaded poles energised by a current transformer whose primary winding is in series with the high-pressure circuit the relay is intended to protect. The torque tending to rotate the pivoted disc is resisted by a weight suspended by a thread passing over a small pulley upon the spindle of the disc. Once the current through the magnet-winding is sufficient to overcome the resisting torque due to the suspended weight, this is raised, and upon reaching its highest position closes the trip coil circuit and operates the oil switch in series with the E.H.P. circuit.

Calibration curves illustrating the action of this type of relay are shown in Fig. 23. The law of the top curve is approximately $T = \frac{A}{c^2} + B$, where T is the time taken by the relay to operate, c is the current passing through the magnet-winding, and A and B are constants. The retarding effect of the copper shoes or "shading" upon the magnetism emanating from the poles is to transform the oscillating field which would be obtained with the poles unshaded into an elliptically rotating field, having a period of revolution identical with the frequency of the magnetising current in the coils of the magnet.

We know that the velocity of a rotating field in the case of an induction motor is given by $\frac{f}{P}$, where f is the frequency of the alternating current, and P is the number of pairs of poles in any one phase.

In the case of the relay under consideration we have an induction motor with one pair of poles. The speed of the rotating field is, therefore, equal to the frequency or f revolutions per second. It would, however, be oscillating in character, and therefore have no effective torque upon the disc if it were not

for the shaded poles. The effect of the copper pole-pieces is to retard the magnetism of this portion of the pole-pieces, so

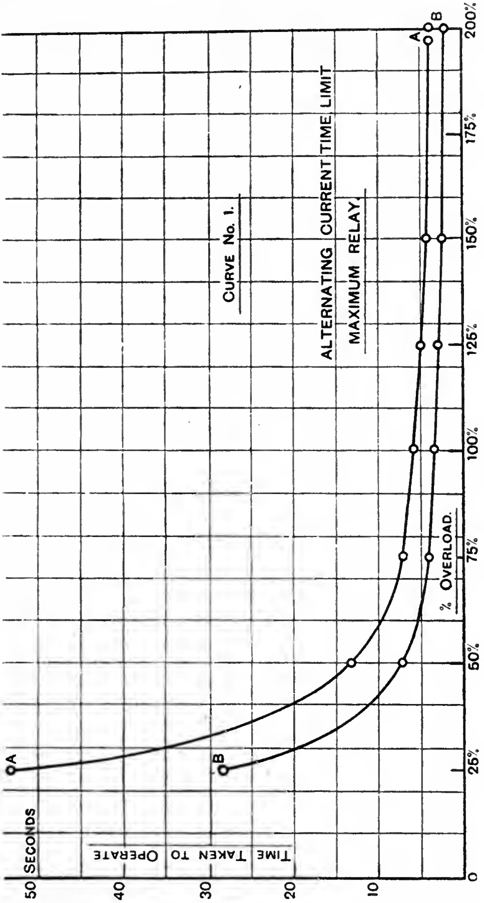


FIG. 23.

that the resultant effect is that of a tuft of magnetism sweeping across the face of the pole-pieces, and dragging the copper disc with it.

It is specially to be noted that the limiting speed which could be attained by the disc would be f revolutions per second, less the slip required to generate sufficient eddy currents in the disc to overcome the retarding forces of the suspended weight and brake magnet.

Now in the case of short circuits heavy currents will pass through the relay magnet. A powerful torque will thus be exerted on the disc momentarily, and it will immediately speed up to reduce the slip. How quickly and nearly the disc approaches synchronous speed will depend upon the extent of the short circuit current, the inertia of the disc and the retarding forces of the weight and permanent magnet.

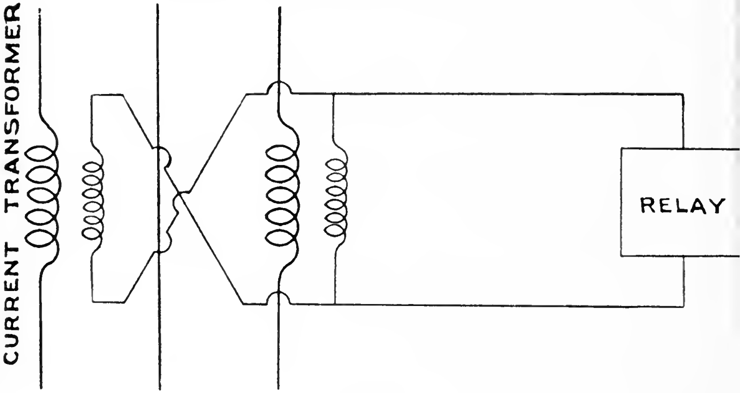


FIG. 24.

Having reviewed the function of the relay itself it becomes of importance to note some special features in the grouping of such apparatus upon switchboards usually carried out.

It may be readily shown that complete protection will only be ensured by installing a separate current transformer upon each phase in circuit with a separate relay.

The arrangement shown in Fig. 24 is sometimes adopted; two current transformers only cross connected being employed in combination with a single relay. This arrangement possesses the following defects:—

If the neutral points of the generators are earthed in ac-

cordance with the usual practice, a fault to earth upon the middle phase could occur without affecting the current transformers and opening the automatic switch.

Moreover, as the current transformers are connected up, the current which must operate the relay is the vectorial sum of the two secondary currents or $\sqrt{3} \times c \times p$ if the current in the secondary winding of each transformer is $p \times c$ amperes at the overload p at which it is required the relay shall operate.

If now an overload occur between one of the phases only in which there is a current transformer, and the phase in which there is no current transformer, a much greater overload must occur before the relay will operate; this overload current being the vectorial resultant of c and $\sqrt{3} \times pc$ drawn as to their correct phase relationship.

Further difficulties are met where polyphase relays or three-phase magnets actuating the same armature or disc are employed. In this instance if a fault occurs between one phase and earth, upon a system where the neutral points of the generators are earthed, a much larger overload on this one phase will generally be required to actuate the mechanism than if the overload occurred simultaneously on all three phases.

We may now proceed to discuss briefly the general working conditions to be met by time element apparatus in practice. All of the devices so far described possess an inverse time element. That is, the larger the overload the smaller will be the time taken by the apparatus to act. Some clockwork relays have been devised, however, to allow a fixed interval to elapse, whatever be the magnitude of the overload, before tripping the switch.

This feature is not really required, but an inverse time element allowing of a predetermined sequence of operation of automatic switches in series. For instance, with feeder switches it is of importance that an overload should cause the distant switches to operate before those at the out-going end of the feeder, which latter should have a time lag sufficient to ensure this result. With relays of disc type this means that the number of revolutions made by the discs before completing the trip coil circuit must differ on the home and distant feeder switches, in other words, the space moved through by the contact mechanism must vary. Under the extreme conditions of a short circuit, however, it is usually difficult to ensure discrimination between more than three relays on the same circuit.

The problem of adequate protection in the case of faults upon the system is further complicated where substations contain moving machinery. In such cases synchronous and induction motors will often feed back through faults necessitating the use of what are known as reverse relays to be described later. With substations containing static transformers and induction motors the outgoing feeder relays are sometimes set to operate in about four seconds, whilst the distant relays are set to operate in two seconds or less, both under the conditions of short circuit. In the case of substations containing synchronous machinery, however, much shorter times will generally be necessary to prevent the synchronous motors falling out of step when a short circuit occurs. No hard and fast rules can be given to suit all cases, and the time element setting of the relays on any particular system should be determined experimentally to suit the special conditions to be met.

In Fig. 23 calibration curves are given illustrating the time element of a disc relay. As a practical example, take the case of a substation containing motor generators connected by a feeder to the generating station. Suppose at the generator end of the feeder a time element relay fixed, also a number of such relays in the substation. A short circuit on one of the motor generators might open the relay switch at the generator end of the line as well as the local relay on the faulty machine.

Suppose the feeder carrying 60 amperes and at the substation supplying three branches of 20 amperes, each branch having its own relay, and each relay set to act in equal times of seven seconds at 50 per cent. overload. The feeder relay will act at 90 amperes. The local relay will act at 30 amperes. At full load the main feeder carries 60 amperes. At full load the subfeeder carries 20 amperes.

Now, a fault upon any one motor generator causing an increase in the current of 30 amperes, or 250 per cent. overload, will cause the local relay to act in about two and a half seconds whilst the feeder relay will operate in seven seconds.

Suppose, however, that a short circuit occurs increasing the feeder current by 90 amperes, or 250 per cent. overload. The relay would operate in two and a half seconds and the local relay would take almost the same time. Therefore, in all probability the main feeder switch will be brought out as well as the local cut out.

If we can separate the horizontal portion of the curves connecting time and overload with the two sets of instruments the less likely is this result to occur. By providing a greater length of cord to be wound up by the feeder relay disc than by the local relay disc before contact is made closing the trip coil circuit, the successive operation of the switches in the desired order may be ensured.

Reverse Relays.—We now come to the consideration of what are known as reverse relays, their function being to impart to automatic oil switches upon alternating current circuits similar features to those possessed by polarised direct current circuit breakers, which open the circuit immediately a current flows in a reverse direction to that required.

Alternating current reverse relays may be classed broadly under two headings:—

(*a*) Reverse Current Relays.
(*b*) Reverse Power Relays.

The usual positions for such apparatus upon alternate current systems are:—

(1) At the far end of feeders supplying substations containing moving machinery or coupled up to other substation networks, which would result in a fault upon the feeder being fed back from the substation itself, causing an extensive disorganisation of the supply unless the local feeder switch be opened immediately.

(2) Between each generator and the main bus bars, to ensure that a faulty generator be at once disconnected from the bus bars and interference with other healthy generators be obviated.

Reverse Current Relays.—As an interesting example of a reverse current relay, many of which might be quoted, may be mentioned an ingenious arrangement devised by Andrews. This consists of a differentially wound solenoid acting upon an iron core, which, when lifted, closes the trip coil circuit. The two windings of the solenoid are connected in parallel across the secondary of a transformer at low pressure, and traverse also what is called an "equaliser," upon which is wound one turn of the main circuit to be controlled (Fig. 25). If no current flows in the main circuit, the low-pressure current divides equally between the differential windings of the solenoid and has no

effect upon the iron core. Should, however, the phase displacement of the current in the main winding be such as would result from a fault, the balance of the currents in the differential

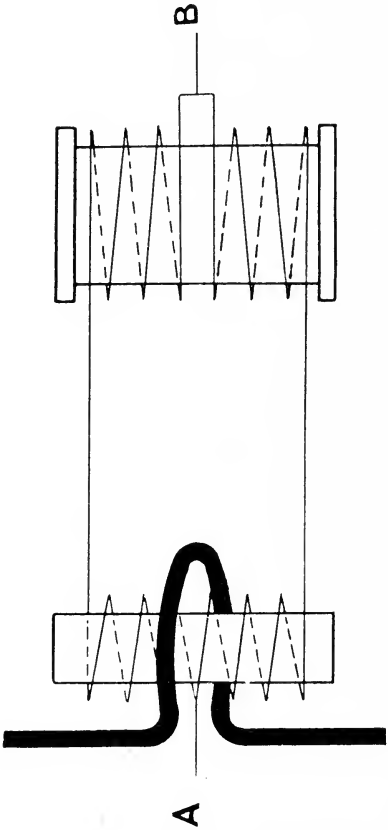


Fig. 25.

windings will be upset, resulting in the core being raised, thus completing the trip coil circuit.

Reverse Power Relays.—The most important of these

devices involve the adoption of the wattmeter principle to a relay originally suggested by Professor Sylvanus Thompson.

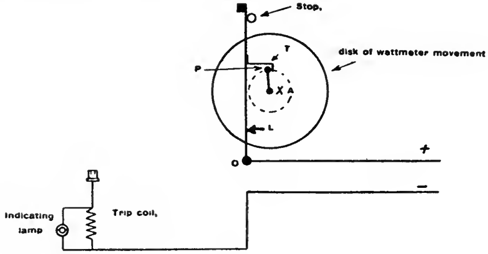
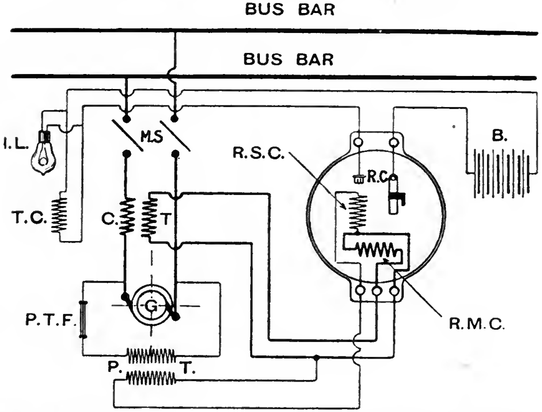


FIG. 26.

Reverse Power Relay Connections.



M.S. = Main Switch.	P.T. = Potential Transformer.
T.C. = Trip Coil.	P.T.F. = Potential Transformer Fuse.
R.S.C. = Relay Shunt Coil.	R.C. = Relay Contactor.
R.M.C. = Relay Main Coil.	B. = Battery or other source of E.M.F. for operating T.C.
G. = Generator.	
C.T. = Current Transformer.	I.L. = Indicating Lamp.

FIG. 27.

When the phase difference of voltage and current are such as to result in the flow of energy in the reverse direction in an alternating current feeder, the relay closes the trip coil circuit.

A compact form of this apparatus is made by Messrs Ferranti, and is illustrated by Figs. 26 and 27.

Fig. 26 explains the manner in which the wattmeter disc, whilst held by a catch T from rotating in a clockwise direction, is free to rotate in a counter clockwise direction, and in doing so will close the trip coil circuit.

One point to be carefully borne in mind with the use of this type of apparatus, depending as it does upon a potential transformer, is that under the conditions of a short circuit the potential difference between the bus bars whether at the sending or receiving end of the line may be so reduced that the apparatus may fail to act.

It is further evident that since the operation of this type of

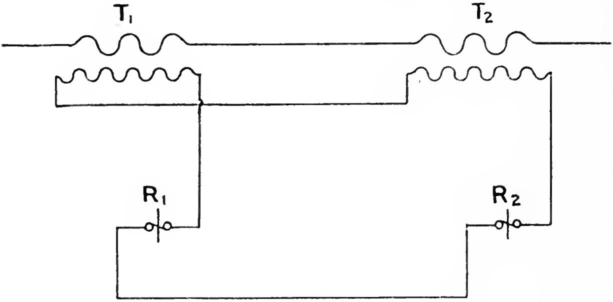


FIG. 28.

relay requires a certain number of watts or real power in the circuit, any reduction in pressure means that an increased current will be necessary. Similarly if the power factor of the circuit be low, although the pressure may be maintained, an increased current will again be necessary to operate the relay. From the preceding remarks it will be gathered that to prevent the reverse current assuming dangerous proportions before the relay operates, it is necessary to strictly limit the reverse power setting. This in the case of relays protecting generators is sometimes made as low as 10 per cent. of the generator output; the power factor of motoring currents being generally low in the case of relays protecting feeders, the reverse power setting may be 20 per cent. or more of the full-rated load of the feeder;

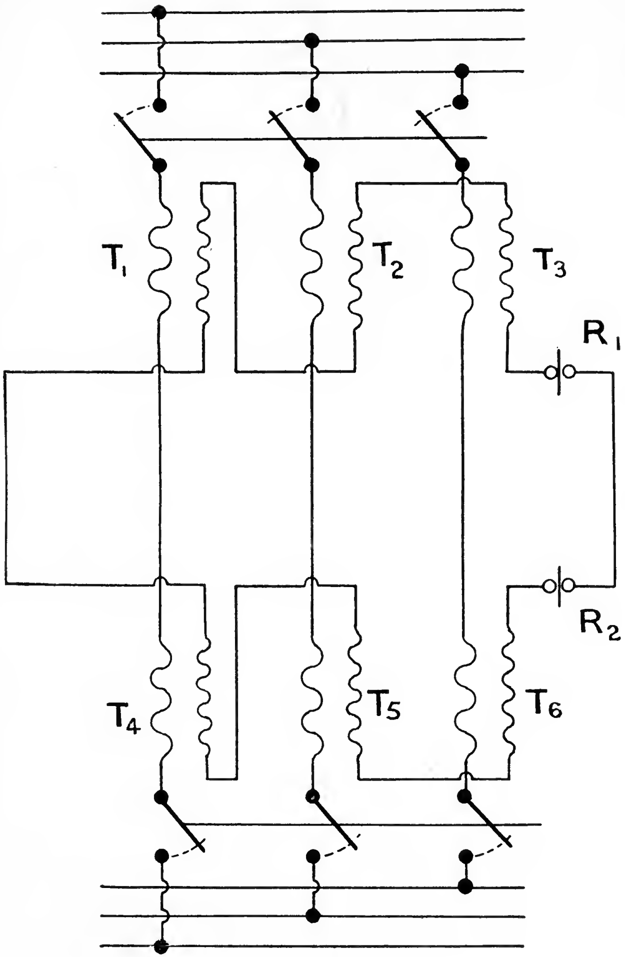


FIG. 29.

the power factor of a fault current will in this case generally be high. It is obvious that if the reverse setting be made too low the hunting of substation machinery or synchronising currents of the generators may open the switches when this is unnecessary. Generally speaking a time element with reverse power relays is undesirable, and when a fault occurs the more quickly it can be isolated from the system the better, and the less likely is the pressure to have fallen to such an extent as to render the relay inoperative.

FIG. 30.

As regards the grouping of reverse power relays the same considerations hold as in the case of maximum time limit relays, and complete protection can only be ensured by employing an independent relay and transformer for each phase.

Merz-Price Protective Gear. — No description of protective switch gear would be complete without some reference to the Merz-Price system, which is extensively used by the Newcastle and Durham Power Companies to protect some hundreds of miles of extra high-pressure cable in networks including feeders, interconnectors, and branch circuits.

The essential points claimed for this apparatus are—

(*a*) The switches do not act in the case of surging or temporary overload.

(*b*) The switches only act when an actual defect arises, and then only on the defective section.

The principle of this protective device will be grasped at once upon reference to Figs. 28 and 29.

Small current transformers, T_1, T_2, are placed at each end of the line with their primary windings in series with it, and the secondary windings are likewise connected in series with one another, and with relays R_1 and R_2, for actuating automatic switches by means of a small auxiliary cable.

As the secondary windings are opposed to one another it will be seen that balance exists for all loads, and that only in the case of a fault on the feeder will a current flow in the relay circuit which will then open the switches at each end of the line simultaneously.

The type of relay used with this gear is illustrated by Fig. 30, and a current transformer of ring type suitable for a 20,000 volt circuit is illustrated by Fig. 31.

FIG. 31.

The same principle is adopted to protect three-phase cables in parallel, banks of transformers, &c.

In applying this system of protection to three-phase cables, a three-core pilot cable is laid with each feeder to form the subsidiary relay circuits, which it is of importance should be insulated from earth to avoid the tripping of the switches on healthy feeders from earth currents set up by a fault.

As one example of the practical utility of this apparatus may be cited that of an interconnector between two substations.

The Merz-Price gear will allow of the flow of energy in either direction between the substations, but upon a fault occurring upon the interconnector itself it is instantaneously cut out of circuit. It is obvious that independent reverse relays could not be employed under such conditions, and that maximum relays at the ends of the interconnector, set high enough to allow of temporary surges of power, could not be arranged to isolate the interconnector instantaneously and before a fault current had reached dangerous proportions, as would be the case with the Merz-Price gear in use.

It is to be noted, however, in connection with this protective gear, that the feeder cables are not protected against ordinary overload such as might be occasioned by the failure of an oil switch in a substation ; its application to an existing system of feeder cables also would be expensive in most cases unless suitable pilot wires were available.

CHAPTER V.

IMPEDANCE, PRESSURE RISE, HARMONICS, &c.

WE have already seen under Chapter II., Lead Sheath Losses, that the impedance of a three-phase, lead-covered, paper-insulated cable is increased by the type of armouring or troughing by which it is surrounded, whereas it is diminished by the effect of the secondary currents induced in the lead sheath. In considering the effect of impedance in a three-core cable it is convenient to deal with one core only. This may to some engineers at first sight present some difficulty from the fact that if we assume a single conductor carrying current and try to calculate its self-induction from the number of lines of force surrounding it when removed from all other conductors, we arrive at a value infinity. The reason is, there must always be a return conductor somewhere, and we will define the self-induction of one core of the cable as half that of the self-induction of two cores, one acting as flow and the other as return ; this will vary directly as the distance apart of the two cores and as the sectional areas of the cores themselves. Now, with a three-phase cable having cores symmetrically spaced, the currents under three-phase working conditions are displaced by a phase difference of $120°$; thus, although when using two cores only of the cable as flow and return half, the E.M.F. of self-induction so measured will give the value per core ; yet when working three-phase the E.M.F. of self-induction measured between phases must be divided by $\sqrt{3}$ to arrive at the E.M.F. of self-induction per core.

The test figures given by Table XVIII. will illustrate this point.

TABLE XVIII.—TEST OF IMPEDANCE OF 6.2 MILES OF (THREE-CORE 0.15 SQ. IN.) EXTRA HIGH-TENSION FEEDER AT 50 ~.

P.D. between Phases at Sending End in Volts.				Current per Core at Sending End in Amps.			Impedance per Single Core in Ohms.
V_1	V_2	Mean.	$\frac{\text{Mean}}{\sqrt{3}}$	C_1	C_2	Mean.	
89	88.5	88	51.2	26	25	25.5	2.01
90	90.5	90.5	52.2	26	25.5	25.75	2.03
99	99	99.5	57.2	28.5	28	28.25	2.02
108	108	108	62.5	31.5	31	31.25	2
116.5	116	116.5	67.2	34	33.5	33.75	1.99
119	119	118.5	68.6	34.2	34	34.1	2.01
						Mean =	2.01

One phase was disconnected and the following readings were taken :—

P.D. between Phases at Sending End in Volts.		Current per Core at Sending End in Amps.			Impedance per Single Core in Ohms.
$V_1 - V_2$	$\frac{V}{2}$	C_1	C_2	Mean.	
92	46	22.5	22	22.25	2.07
100	50	25	24	24.5	2.04
99.5	49.75	25.5	24	24.7	2.02
110.5	55.25	27.5	26.5	27	2.04
122	61	31.2	30.5	30.8	1.98
123.5	61.75	31.5	30.7	31.1	1.99
140	70	35.7	34.5	35.1	2
141	70.5	35.8	35	35.4	1.99
				Mean =	2.01

It is useful at this point to note the following items :—

Resistance due to self-induction $= 2\pi n\mathrm{L} = p\mathrm{L}$ ohms,

E.M.F. due to self-induction $= 2\pi n\mathrm{LC} = p\mathrm{LC}$ volts,

where $n =$ frequency.

$\mathrm{L} =$ self-induction in henrys per conductor.

$\mathrm{R} =$ resistance in ohms per conductor.

$c =$ current in amperes per conductor.

Impedance of conductor = Resultant ohms $= \sqrt{\mathrm{R}^2 + p^2\mathrm{L}^2}$.

In the case of low-pressure distributors of three and four core type, the impedance may often become of importance by

Induction per Core per Mile at 50 ~ of Paper-Insulated E.S.C. Three-Core Cables Constructed for Various Working Pressures.

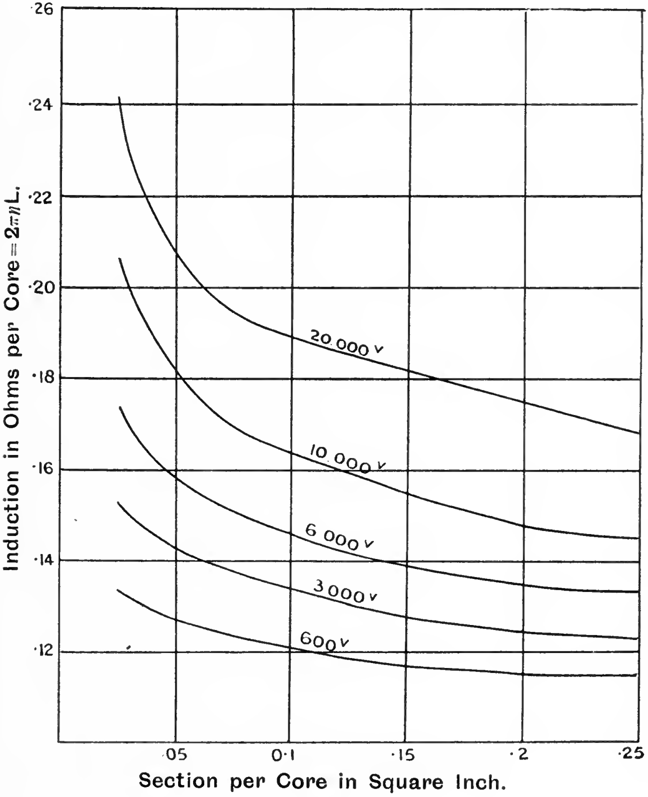


Fig. 32.

increasing the drop in pressure over that due to resistance alone by as much as 20 per cent. or more. With high-pressure

cables, although this impedance may exceed the copper resistance by as much as 40 to 50 per cent., the drop in pressure will in general be but a small percentage of the transmission pressure, and, therefore, of little importance.

In Figs. 32 and 33 curves are given illustrating the induction effect per core of paper cables constructed according to the Engineering Standards Committee's specification as regards thickness of dielectric, &c., for the working pressures stated.

It is interesting to note that with a working pressure of

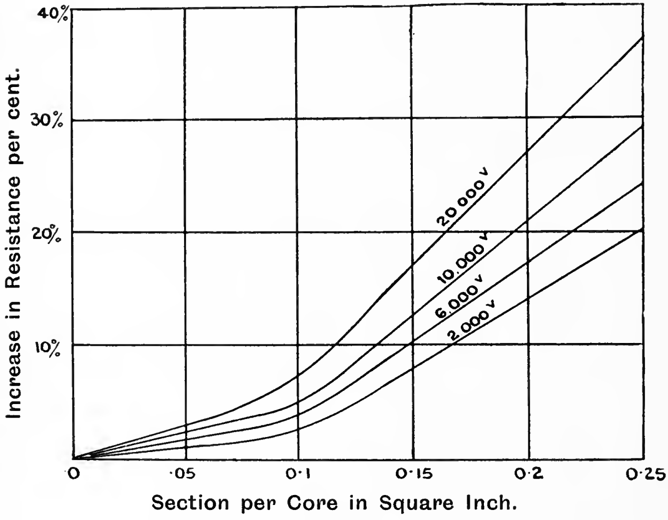


FIG. 33.

20,000 volts the impedance per core of a 0.25 sq. in. three-core cable exceeds the copper resistance by nearly 40 per cent.

In the case of four-core cables used for three-phase working, one of the conductors, usually termed the neutral, will carry the out-of-balance current only. The presence of this fourth or neutral conductor disturbs the symmetrical spacing of the live conductors since one of these will have an active conductor upon each side of it, whereas the two remaining live conductors will have an active conductor on one side of them only. The

following test on a drum containing 220 yards of 0.15 sq. in. L.T. four-core British Insulated & Helsby Cable Co.'s lead-covered cable will illustrate this point.

Star connected transformers were joined to one end of the cable, and at the other end three coils of $\frac{7}{20}$ V.I.R. cable (braided only) were joined in star.

Each of the three conductors was joined consecutively to an oscillograph to record the current flowing in it, and the drop in pressure along the conductor.

The results obtained are given in Table XIX.

TABLE XIX.

	Core of Cable.			
	Red.	Blue.	Green.	Neutral.
(1) Amperes - - - - -	12.5	11.50	11.90	0.00
P.D. along conductor -	.568	.460	.625	...
Impedance $=\frac{V}{C}$ - - -	.045	.040	.052	...
Direct current $\frac{V}{C}$ - -	.0387	.0388	.0386	...
(2) Amperes - - - - -	13.6	9.87	12.27	5.0
P.D. along conductor -	.642	.384	.568	...
Impedance $=\frac{V}{C}$ - - -	.047	.0389	.0477	...
Direct current $\frac{V}{C}$ - -	.0387	.0388	.0386	...

From the above table it will be seen that the impedance for the blue conductor, which has an active conductor on each side of it, is small, whereas the red and green conductors, which have an active conductor on one side only, have an impedance of about 20 per cent. in excess of that with direct current. The current in the red and green conductors had a considerable lag, whereas that in the blue conductor had but a small lag.

Pressure Rises.

The principal causes of pressure rise met with in transmission circuits may be classed as follows :—

(1) Resonance.
(2) Power surges.
(3) Reflected waves.
(4) Concentration of potential.
(5) The effect of leading currents on generators and transformers.

In what follows it is proposed to discuss briefly these effects.

Resonance.—In acoustics we have very familiar cases of resonance. As examples, a great number of which might be mentioned, we may note the singing of violin strings when in the neighbourhood of a piano upon which notes are struck in sympathy with the period of vibration of the string; the same effect occurs with gas globes or other objects in a room in which musical sounds are being produced. This phenomenon of resonance, which we find present in all the principal sciences, is very much in evidence in alternating current circuits. To take a mechanical analogy, we may suppose a heavy weight w attached to a spiral spring which we know will possess a natural frequency of vibration and which we may denote by the letter n. If we now impart to the point of support a forced vibration of frequency m it may be readily shown that the amplitude of the vibration A of the weight w will be :—

$$A = \frac{1}{1 - \frac{m^2}{n^2}} \text{ times that of the point of support.}$$

If n equal m we have an amplitude theoretically infinity.

It is of great importance to notice from the above formula that when the motion of the point of support is a small fraction of the natural frequency, the forced vibration of the weight is practically a copy of the motion of the point of support; in other words, the spring and weight w move like a rigid body, but as the frequency of the forced vibration gets more nearly equal to the natural frequency of vibration of the weight on the spring the amplitude of its motion is enormously increased. It will also be noticed that when the forced vibration is made many times the frequency of the natural vibration of w, the

amplitude of w becomes less and less. This result we know by experience to be the case. An interesting piece of apparatus based upon this fundamental principle has been developed for determining the frequency of an alternating current. The construction of this apparatus is shown in Figs. 34 and 35, and is known as Frahm's frequency indicator. As will be seen, it consists of a number of vibrating tongues, such as we find in an ordinary musical box, each of which is weighted and of sufficient length to have a natural frequency of vibration to correspond with the natural frequencies of those alternating current circuits usually to be met with in practice. The alternating current whose frequency is to be determined causes the point of support of these tongues to vibrate by means of an electro-magnet through which the alternating current passes,

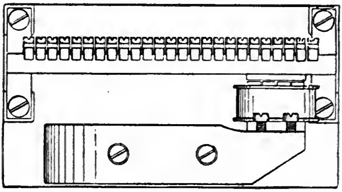


FIG. 34.

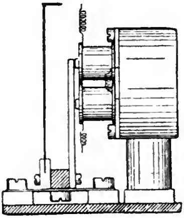


FIG. 35.

when one or more of the tongues will be set in violent oscillation, according as its natural frequency of vibration is in sympathy with that of the point of support.

It is shown in text-books that if we have a circuit consisting of a self-induction of L henrys in series with a capacity of K farads, as in Fig. 36, the circuit will possess a natural frequency for electrical oscillations ; this frequency, N, being given by the expression :—

$$N = \frac{1}{2\pi}\sqrt{\frac{1}{LK}} \quad - \quad - \quad - \quad - \quad (1)$$

If in this circuit be included an alternator of voltage V and frequency N, we have the conditions necessary for electrical resonance and the amplitude of the current C will become very large, depending only upon the ohmic resistance of the conductors and alternator ACB in Fig. 36.

The necessary conditions for resonance may be stated in another way. If the self-induction L be alone in circuit, the current C_L would be

$$C_L = \frac{V}{2\pi NL} \text{ amperes} \qquad - \quad - \quad - \qquad (2)$$

Similarly, if the condenser were alone in circuit the current C_K would be

$$C_K = 2\pi NKV \text{ amperes} \qquad - \quad - \quad - \qquad (3)$$

When $C_L = C_K$ we have the necessary condition for resonance, for equating (2) and (3) we get

$$(2\pi N)^2 = \frac{1}{LK}, \text{ or } N = \frac{1}{2\pi}\sqrt{\frac{1}{LK}} \text{ as before.}$$

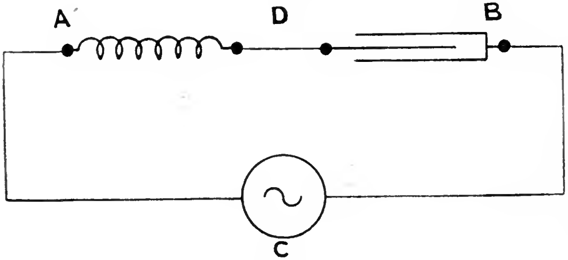


FIG. 36.

We know also that for any current C traversing the circuit the pressure across the self-induction will lag 90° behind this current, whilst the pressure across the condenser will be 90° in advance of this current. These two pressures will be thus equal and opposite at every instant. Hence, when resonance occurs the effect is the same as if AB were short-circuited. The current C is then only limited by the resistance of the circuit ACB.

If C^1 = self-induction current or condenser current when one or the other is alone in circuit.

$\frac{C}{C^1}V$ = pressure across A D or D B, and this may reach a highly destructive value.

The rise of potential within the circuit being given by $C\sqrt{\frac{L}{K}}$.

It is to be noted at this point that harmonics present upon the fundamental pressure wave are in general quite free to act as separate entities in producing resonance. In fact with serious resonance of harmonics the fundamental wave may become almost indistinguishable, being replaced by a wave of great amplitude with a frequency as many times that of the fundamental as corresponds to the order of the particular harmonic for the resonance of which the conditions of the circuit are favourable. Let us take a particular case. A star wound 500-kw. 5,000-volt three-phase generator was found to have an average impedance, due to self-induction, of 34.5 ohms per phase at a frequency of 50 cycles, corresponding to a short circuit current of 145 amperes at 5,000 volts; 10 miles of three-core 0.15 sq. in. 5000-volt cable will take a minimum charging current of 2.96 amperes per phase.

Now suppose a strong 7th harmonic present in the pressure wave of the alternator. The charging current due to this harmonic will increase, whereas the current through the self-induction will diminish, in proportion to the frequency. The short-circuit current through the alternator, assuming constant self-induction, will be $\frac{145}{7} = 20.7$ amperes. The charging current in the cable will be $2.96 \times 7 = 20.7$ amperes. It will, therefore, be seen that we have the necessary condition for resonance of the 7th harmonic.

Variations in the forms of pressure waves due to conditions such as the above are discussed later when dealing with wave forms.

It is to be noted, however, with regard to triple frequency harmonic currents and multiples that the generator impedance to these will not be three, or multiples of three, times that due to the fundamental wave, for the reason that the phase relationship of these currents in the windings of the alternator will produce opposite magnetising effects upon the iron, and thus the impedance to these higher harmonic currents will not be strictly proportional to their frequency. In general a capacity considerably in excess of that theoretically necessary, as determined by the above considerations, is required to produce exact resonance, a further error is also introduced by assuming that the self-induction of the alternator, as deduced from the value of its short-circuit current or synchronous impedance, will be the same

for all loads and currents. This will be obvious upon considering the shape of the synchronous impedance curves of most generators.

It may be gathered from what has preceded that resonance at fundamental frequency will not generally occur with commercial alternators and such cable systems as are usually fed by same, owing to the fact that the self-induction of the alternator is generally too small, and that this is further diminished in proportion to the number of similar alternators running in parallel.

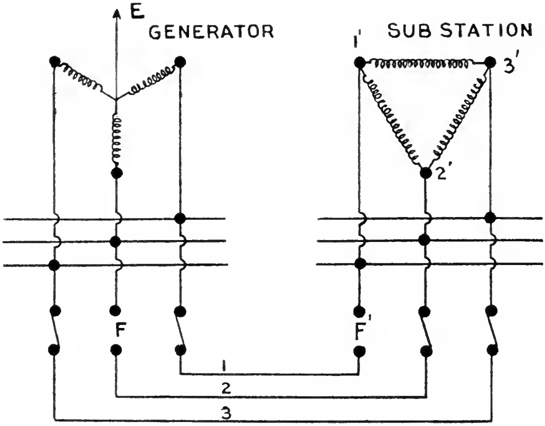


FIG. 37.

Certain conditions may arise, however, in practice which will result in resonance at the fundamental frequency of the system.

As an example we may take the case of a substation containing three 100 kw. delta connected transformers fed by a 0.4 sq. in. three-core cable at 5,000 volts, 50 cycles, from a distributing centre one mile distant.

The magnetising current of each transformer may be taken at 0.183 ampere, corresponding to a primary impedance of 27,320 ohms or a self-induction of 87 henrys.

The core capacities of the cable may be taken as follows:—

One core *versus* two others bunched to lead sheath = .26 microfarad.
Three cores *versus* lead sheath = 0.5 microfarad.

Now, consider the state of things illustrated by Figs. 37 and 38, where one phase is shown open at the distributing station and another phase open at the substation, a condition which might be brought about by the blowing of fuses F F_1 upon an overload or other cause.

Current will pass from the generator *via* core No. 3 of the cable to the transformer terminal 3′, then *via* transformer winding 3′-2′ and also *via* windings 3′-1′ in series with 1′-2′ to core No. 2 of the cable. From core No. 2 this current will pass

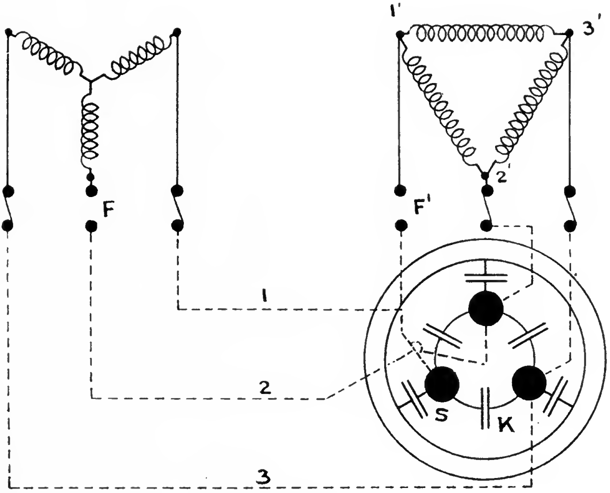


Fig. 38.

through the condenser formed by the dielectric between cores 2 and 1, and *via* core No. 1 back to the generator.

The capacities between the cores of the cable and between cores and lead sheath are equivalent to equal condensers K and S arranged as shown in Fig. 38.

The capacity between one core and two others bunched to the lead sheath is obviously 2 K+S, whilst the capacity between all three cores bunched together and the lead sheath is 3 S′.

From the data already given, therefore, we have

$$2K + S = 0.26 \text{ microfarad.}$$

$$3S = 0.5 \quad ,,$$

from which we deduce $K = .045$,,

and $S = 0.17$,,

Now the combined self-induction of two transformer windings in series shunted by a third transformer winding, each winding having a self-induction of 87 henrys, is obviously $\frac{2}{3} \times 87 = 58$ henrys. Our circuit, therefore, resolves itself into reactive branches consisting of a self-induction of 58 henrys shunted by a capacity of .045 microfarad, this combination being in series with a capacity of 0.13 microfarad (Fig. 39).

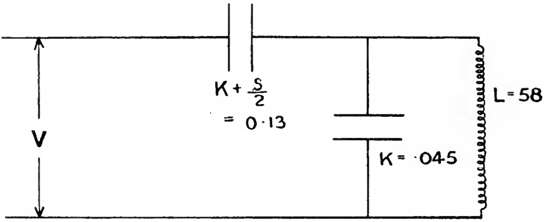


FIG. 39.

The reactance of the condenser and self-induction in parallel will be:—

$$\frac{L}{1 - p^2LK} = \frac{58}{1 - (314)^2 \times 58 \times .045 \times 10^{-6}} = 78 \text{ henrys.}$$

which may be considered as in series with the condenser of 0.13 microfarad.

But the condition for resonance is that K shall be equal to

$$\frac{1}{p^2L}, \text{ or } K = \frac{1}{(314)^2 \times 78} = \frac{0.13}{10^6}.$$

We see, therefore, that the necessary condition for resonance is satisfied in this instance at the fundamental frequency of the circuit, and highly destructive pressure rises have been found to occur under the conditions set out above.

Power Surges.—The most important rises of pressure upon transmission lines are usually those due to power surges, such as are obtained when a short circuit occurs upon the line, largely

increasing the current, which is subsequently interrupted. The surge pressure in such cases has been found to depend almost entirely upon the magnitude of the current flowing, and to be independent of the length of the line and its working pressure.

If we have a circuit made up of a large condenser K (Fig. 40) in series with a self-induction L, and suppose the condenser to be charged to a high potential, we know that on closing the switch S the condenser will discharge itself through the self-induction, the current increasing until the condenser is fully discharged. At this instant the whole of the potential energy stored in the condenser is now stored in the self-induction in the form of electro-magnetic energy. The current will not cease to flow abruptly, but will continue in the same direction due to the E.M.F. of the self-induction until the condenser is

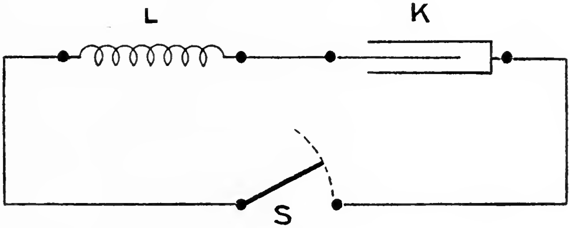


Fig. 40.

again charged, but with reversed polarity to that originally. The condenser again discharges into the self-induction, and the current oscillates until the energy of the charge is used up in the heating of the circuit.

The energy stored up by a condenser of K farads charged to a potential of V volts in watt-seconds is given by the expression $\frac{1}{2}KV^2$. Again, the energy stored up in the magnetic field of a self-induction of L henrys, when carrying a current of C amperes in watt-seconds, is given by the expression $\frac{1}{2}LC^2$, and this same amount of energy may appear alternately as potential or electro-magnetic in a precisely similar manner to the energy of the bob of a swinging pendulum, which is wholly potential at the end of its swing, and wholly kinetic as it passes the lowest point of its swing.

In the case of long transmission lines, or extensive networks

of extra high-pressure cables, we have considerable capacity and self-induction, and when such circuits are carrying heavy currents at the instant they are interrupted, the total energy stored electro-magnetically in the field surrounding the conductors of the circuit will appear as potential energy, charging up the circuit to a very high voltage.

Taking as an example the case of an overhead transmission line, we may assume the self-induction L of one line wire per mile to be .0037 henry, and its capacity 0.8×10^{-8} farad. If a short circuit occurred, causing the interruption of the line when the current had an instantaneous value of 150 amperes, we get for the value of $\frac{1}{2}LC^2$ the electro-magnetic energy of the circuit:—

$$\frac{.0037 \times (150)^2}{2} = 42.4 \text{ watt-seconds.}$$

Now upon the interruption of the line this energy must appear as a charge increasing the potential of the circuit; in other words

$$\tfrac{1}{2}KV^2 = 42.4$$

or

$$V = \sqrt{\frac{42.4 \times 2 \times 10^8}{.8}}$$
$$= 103{,}000 \text{ volts.}$$

Experiments made in America upon a line 100 miles in length, short circuited at the far end, resulted in a power surge which jumped spark gaps $4\frac{1}{2}$ in. in width at the generator end of the line. Allowing 24 kilovolts as an ordinary value of the dielectric strength of air, this sparking distance would in itself correspond with an instantaneous pressure rise of 108,000 volts.

It will be found that in cable systems the pressure rises are not anything like so great under the same conditions. For instance, in the case of a 20,000-volt .05 sq. in. three-core cable we may take $L = .748 \times 10^{-3}$ henry per mile, $K = .211 \times 10^{-6}$ farad per mile, and assuming the current interrupted to have an instantaneous value of 150 amperes as before, we get—

$$\tfrac{1}{2}LC^2 = \frac{.748 \times (150)^2}{2 \times 10^3} = 8.4$$

and

$$V = \sqrt{\frac{8.4 \times 2 \times 10^6}{.211}} = 8{,}940 \text{ volts.}$$

In practice power surges are very much more complicated than might be assumed from the above theoretical considera-

tions, and in general the voltage rise due to a surge will be superposed upon the working pressure of the line.

It will thus be seen that a line working at a very high pressure, but with a small current, may not be subjected to so high a surge pressure as a line at lower working pressure but having a much larger line current.

When a transmission line is interrupted, it may be shown that a pressure oscillation will be started of maximum value V, which will be slightly less than that given by the expression :—

$$V=\sqrt{V_1^2+C_1^2\frac{L}{K}}$$

Where V_1 = instantaneous value of line pressure at time of interruption.
C_1 = instantaneous value of line current at time of interruption.
L = self-induction per mile per conductor in henrys.
K = capacity per mile per conductor in farads.

Take the following examples :—

	LINE PRESSURE. Volts.	PRESSURE TO EARTH. Volts.	LINE CURRENT. Amperes.
(*a*) - - - - -	60,000	48,000	50
(*b*) - - - - -	30,000	28,000	100

And values of L and K :—

L = .00197 henry per mile.
K = .0153 microfarad per mile.

If we assume that the current is doubled by a short circuit at the time of the line being interrupted, and that the instantaneous value of the line pressure is equal to the effective pressure, then we get for the first case :—

$$Va=\sqrt{(48000)^2+1288\times10^6}=60{,}000 \text{ volts};$$

and for the second case :—

$$Vb=\sqrt{(28000)^2+5152\times10^6}=77{,}000 \text{ volts}.$$

It is of interest to compare the above results with the surge pressures likely to be set up upon an underground cable system.

Take, for example, a three-core 20,000-volt cable—

	LINE PRESSURE. Volts.	PRESSURE TO EARTH. Volts.	LINE CURRENT. Amperes.
(*c*) - - - - -	20,000	11,550	75

Normal values for L and K in this case are:—

$L = .748 \times 10^{-3}$ henrys per conductor per mile.
$K = 0.211$ microfarad Y capacity per conductor per mile.

Assuming that the working current is doubled at the moment of interruption, and that the value of the pressure V_1 is at its maximum, we get for Vc—

$$Vc = \sqrt{(16330)^2 + 79.8 \times 10^6}$$
$$= 18,650.$$

The insulation between cores and lead sheath would thus be subject to an instantaneous excess pressure of $18,650 - 16,340 = 2,310$ volts under the conditions assumed.

Reflected Waves.—If A B, B C, and C D (Fig. 41) represent open troughs containing water, we know that if a disturbance at

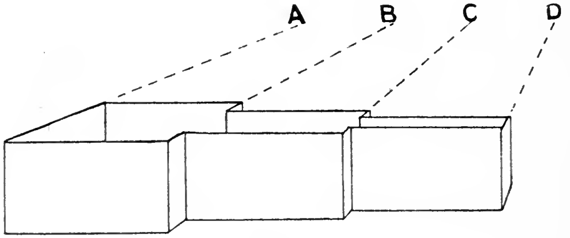


FIG. 41.

C cause a wave to proceed in the direction C D, this would, on reaching D, be reflected, the amplitude of the wave being increased to double its original value.

Suppose a similar disturbance to take place at A, causing a wave to travel in the direction A to B, upon reaching B the smaller trough B C will be unable to absorb the wave, resulting in an increase of its amplitude which may be nearly twice that of the original wave. A wave of increased amplitude now traverses the trough B C, and is similarly increased in amplitude to nearly twice its value at the point C. A wave of four times the amplitude of the original disturbance will now traverse the trough C D, and will finally be reflected at the closed end D with an amplitude approaching eight times that of the original wave.

A transmission line may be considered as made up of a

number of reactances and capacities arranged as illustrated by Fig. 42.

If the switch is closed, applying a high voltage to the cable instantaneously, the current will be retarded by the back E.M.F., due to the building up of the field round the cable cores, and the charging up of the cable will be accomplished gradually through the reactance of its cores.

The current will increase in value and reach its maximum at the instant the cable is charged up to the applied pressure. The charge having reached the open end of the line the current tends to stop, but the energy stored in the magnetic field of the cable due to its self-induction (corresponding to kinetic energy in a moving body) now tends to keep the current flowing, which

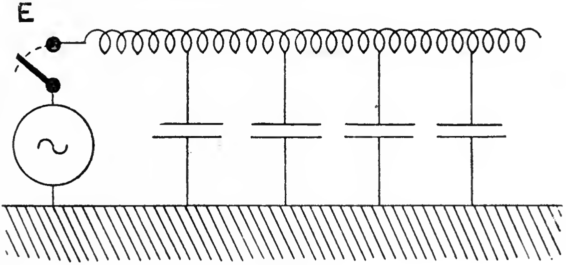


FIG. 42.

continues to charge up the open end of the cable to double the applied pressure or thereabouts.

A wave of double pressure is then reflected along the cable, and the potential of the cable will oscillate, as shown by Fig. 43, until it gradually settles down at line potential.

If the self-induction per mile of one core of a transmission line or cable is "l" henrys, and the capacity per mile of core is "k" farads, it may be shown that the quantity $\sqrt{\frac{1}{kl}}$ represents the velocity in miles per second with which an electrical disturbance would be propagated along the core, this velocity being so great that the resistance of the cable may be neglected.

When such a cable is switched on to an alternator, electrical impulses pass into the cable and are reflected at its open end. Should it happen that such impulses return to the generator end

of the cable in the time represented by one half period of the generator wave, we get conditions favourable to resonance, since the reflected waves would then be in phase with the generator waves. In other words, if the time required by the impulses to travel four times the length of the cable corresponds with the periodic time of the alternator, the pressure will increase due to resonance until a breakdown of the insulation occurs.

As an example, take the case of a network having 80 miles of .05 three-core 20,000-volt cable, we may assume—

$$l = .748 \times 10^{-3} \text{ henrys per mile;}$$
$$k = .211 \times 10^{-6} \text{ farads per mile.}$$

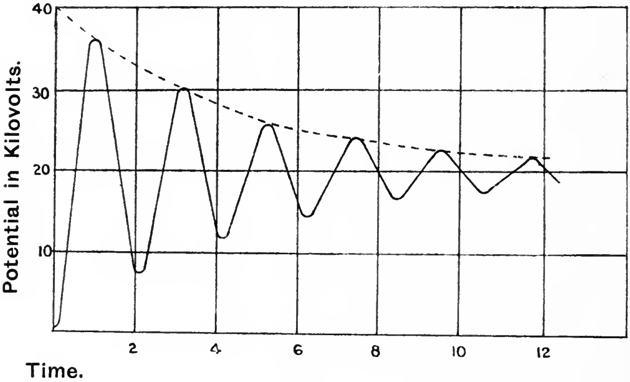


Fig. 43.

The velocity of propagation will be

$$\sqrt{\frac{1}{kl}} = 79{,}600 \text{ miles per second,}$$

and the frequency of the natural period of oscillation is

$$\frac{79600}{80 \times 4} = 250 \text{ per second.}$$

Hence, with an alternator giving a pressure wave of 50 cycles per second, and having a pronounced 5th harmonic impressed upon it, we should expect to get resonance and a serious rise of pressure from this effect.

A further phenomenon of pressure rise or concentration of potential is very much in evidence on most systems working at extra high pressure. If we have a loop of copper wire shaped as in Fig. 44, it is well known that a static discharge passing from A to B will jump across the air-gap in preference to traversing the metallic shunt to the air-gap formed by the wire. The reason being that a self-induction behaves almost as a complete insulator when subject to such extremely rapid oscillations of pressure as occur with static discharges. In the case of transformers or motors switched on to high-pressure lines, the pressure at the instant of switching on is concentrated between the first few turns of the windings. Each of these

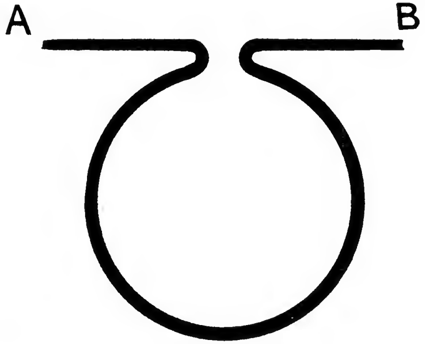


Fig. 44.

turns has a self-induction, a capacity to the next turn and a capacity to earth (Fig. 45), and until these capacities become charged up, which process is delayed by the very large self-induction effect resisting the charge, the adjacent turns or layers of the windings upon the end coils may have to withstand the full potential of the circuit concentrated between them.

It is to be noted that precisely the same effect will result if one terminal of the transformer or motor becomes suddenly earthed through a fault. The end turn in this case will be instantly brought to earth potential, but the self-induction of the adjacent turns will resist the immediate change of condition resulting in a high potential gradient across the insulation.

Various remedies have been suggested for reducing this

effect, probably the simplest and most effective being to provide choking coils external to the motor or transformer terminals to take the shock at switching on, &c.

In the case of E.H.T. transformers also there is another effect met with, although of comparatively rare occurrence. This is the current rush which sometimes occurs upon switching the transformer into the circuit, resulting in a pressure rise, and is generally explained by the supposition that the magnetic condition of the iron at the instant of switching off was retained, and of such sign and value as to greatly assist instead of impede the primary current at the instant of switching on. In the case

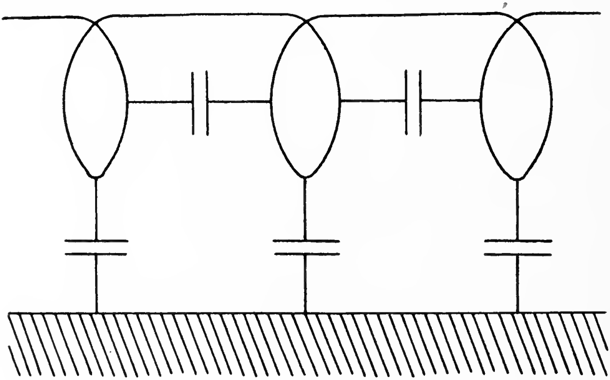


Fig. 45.

of motors, however, the slowing down of the rotor generally tends to wipe out any residual magnetism in the stator.

Wave Forms.—A conductor revolving with uniform velocity in a uniform magnetic field will have a pressure wave of true sine form impressed upon it, and if we plot on squared paper its values at equal intervals we have a representation of what is known in mechanics as a pure harmonic motion. Such a curve would be traced by a pencil attached to the bob of a heavy pendulum under which a piece of drawing paper was moved with constant velocity at right angles to the direction of swing of the pendulum. It is, however, rarely in practice that a pressure wave of this form is obtained, and even in cases

where an alternator develops a true sine pressure wave, the resultant current wave will often have a form which is considerably different from that of the pressure wave owing to the fact that ordinary circuits in practice, such as those containing arc lamps and other apparatus, do not possess constant resistance or reactance. In recent years effective steps have been taken in the design of alternators to produce pure sine waves by the shaping of pole-pieces on the rotors, and the angular spacing of the conductors upon the stators to counteract the effects of irregularities in the magnetic field due to the stator slots. Fig. 46 shows the form of pressure wave of a generator having six stator slots per pair of poles. The equation of the curve shown in this figure may be expressed in the form :—

$$V = A_1 \sin(\theta + \phi_1) + A_3 \sin(3\theta + \phi_3) + A_5 \sin(5\theta + \phi_5), \text{ \&c.}$$

Where A_1, A_3, A_5, &c., are the amplitudes of the fundamental wave, 3rd, 5th, &c., harmonics, respectively, and ϕ_1, ϕ_2, ϕ_3, &c., give the phase displacements of these harmonics. The values of A_1, A_3, &c., ϕ_1, ϕ_3, &c., for the wave form shown in Fig. 46, are given in the following table :—

TABLE XX.

Order of Harmonic.	Amplitude in Volts=A.	Phase Displacement in Degrees=ϕ.
1st - - -	6,750	...
3rd - - -	148	156
5th - - -	1,190	171
7th - - -	45	75
9th - - -	18	27
11th - - -	418	166
13th - - -	185	131
15th - - -	97	108
17th - - -	270	23

It will be noticed that the most important harmonics are the 5th, 11th, and 17th. In general, if there are p slots in the stator of the generator per pair of poles, the most important harmonics to be expected are the $(p-1)$th or the $(p+1)$th.

If we plot a sine wave and impose on it an even harmonic (Fig. 47), it is seen that the two halves of the resulting wave

become dissimilar, and we deduce from this that even harmonics will not be present in the pressure waves of commercial alternators. If upon the fundamental wave we impose a third

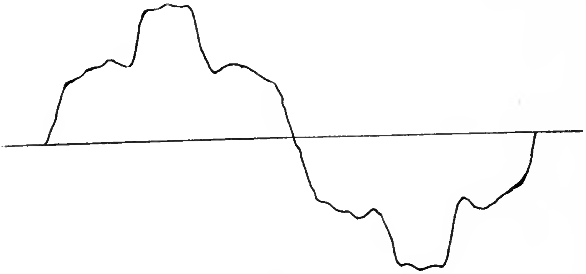


FIG. 46.

harmonic or any harmonic of a multiple of three times the fundamental frequency, the three pressures, one in each phase, become superposed, that is, they all act in the same direction at

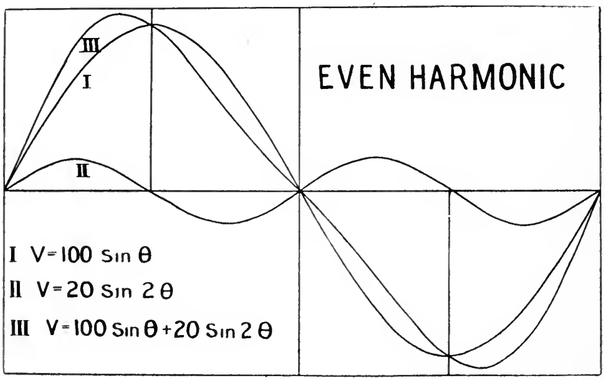


FIG. 47.

the same instant in each phase. This result is of great importance in practice. For instance, with delta-connected alternators and a pressure wave possessing triple frequency harmonics, idle

currents will circulate round the closed mesh formed by the three delta-connected windings.

Again, if two or more star-wound alternators with earthed neutral points are worked in parallel, the pressure wave of one of them possessing a triple frequency harmonic, whilst those of the others are practically sine waves, heavy triple frequency currents may result, flowing between the neutral points and the phase windings of each machine. In order to avoid this, the practice of earthing one machine only in the station has sometimes been adopted.

The writer took oscillograms of such currents on a three-phase system, and found that they varied throughout the day with the nature of the load on the station, and according to the capacity of the cables and self-induction of motors, &c., on the circuits. It was found that capacity currents of frequency 3, 6, 9, &c., times the fundamental wave were flowing between the neutral points of the generators and earth. An account of some observations made upon these earth currents will be found in the *Journal of the Institute of Electrical Engineers*, Vol. xxxiii., Part 166.

The complete analysis of any irregular wave form may be readily effected by a number of well-known methods. Space will only permit here of the description of the following simple and approximate method.

Divide the complete wave from 0 to 360°, or both positive and negative portions, into the same number of parts as the order of the harmonic it is desired to examine. That is, if we wanted to determine the amplitude of the 3rd harmonic with reference to the fundamental wave, we should divide the complete wave into three parts, and again subdivide the zero line of each of these three parts into the same number of equal divisions. Add the ordinates of each of the three parts at corresponding divisions of the zero axis, and divide the result by 3 for the third harmonic, 5 for the 5th harmonic, and so on. These results will give points upon the harmonic curve which can then be plotted on the axes of the pressure waves under examination. The reason for this procedure is obvious, since the addition of the ordinates in a pure sine wave divided into equal parts at corresponding points would always be zero, whereas the ordinates of the harmonics would have equal values at corresponding divisions in the equal parts positive and negative, and hence the reason

8

for dividing the sum of the ordinates by the order of the harmonic or number of subdivisions of the wave. As an example, take the pressure wave illustrated by Fig. 48.

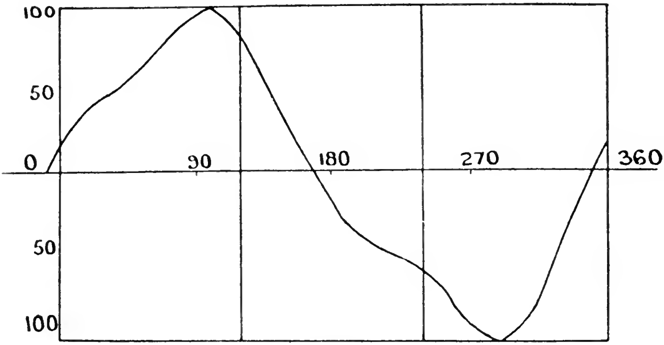


FIG. 48.

To determine the amplitude of the 3rd harmonic, we divide the curve into three parts as shown. We then superpose these three divisions (Fig. 49), and add corre-

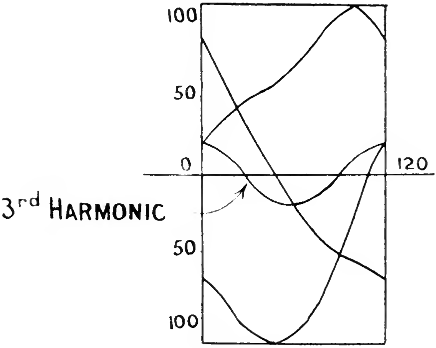


FIG. 49.

sponding ordinates, dividing each sum by 3, the result gives the ordinate of the 3rd harmonic at each of the respective points. It is to be noted, however, that the curve so obtained

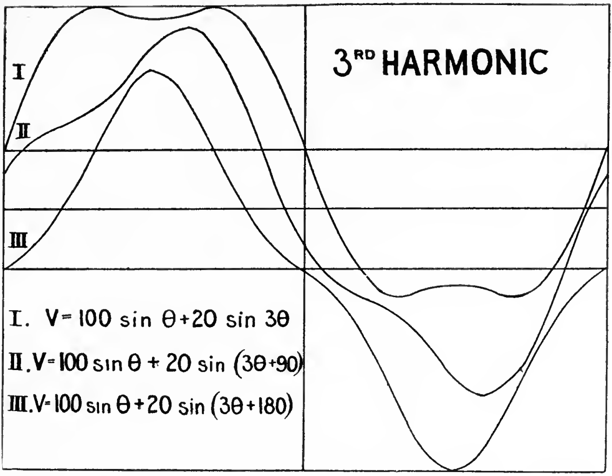

3RD HARMONIC
I
II
III
I. V = 100 sin θ + 20 sin 3θ
II. V = 100 sin θ + 20 sin (3θ + 90)
III. V = 100 sin θ + 20 sin (3θ + 180)

FIG. 50.

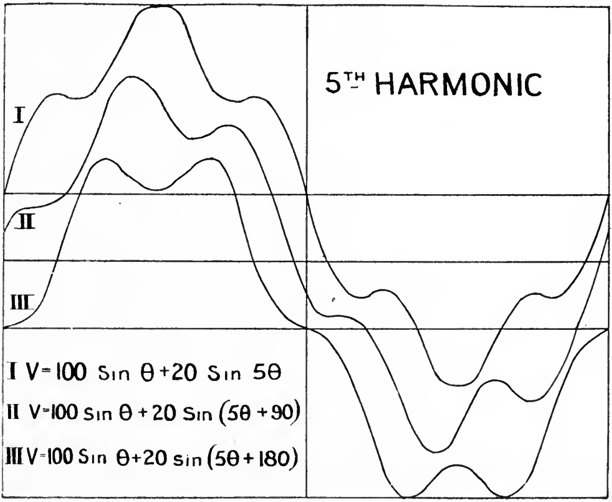

5TH HARMONIC
I
II
III
I V = 100 Sin θ + 20 Sin 5θ
II V = 100 Sin θ + 20 Sin (5θ + 90)
III V = 100 Sin θ + 20 sin (5θ + 180)

FIG. 51.

may not be a true sine curve. The variation in wave form due to any particular harmonic as its displacement relatively to the fundamental wave is increased, is shown by Figs. 50 and 51. The successive curves are obtained by the displacement of a third and fifth harmonic with an amplitude of one-fifth of the fundamental by 90° and 180° respectively. It is evident that, according to the displacement of the harmonic relatively to the fundamental wave, the maximum instantaneous pressure may be increased or diminished. This brings us to the con-

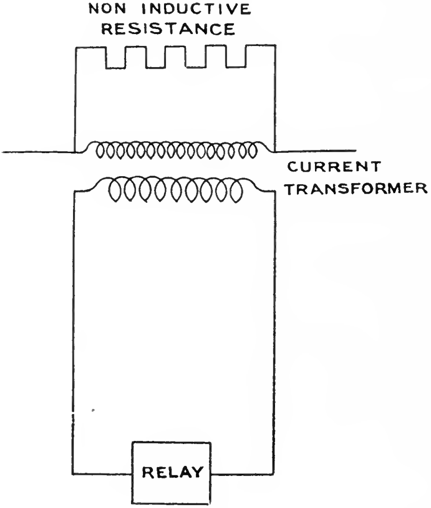


Fig. 52.

sideration of what is known as the form factor of a pressure wave.

The usually accepted definition of the form factor of a pressure wave is the ratio of the maximum ordinate to the square root of the mean square of the ordinates.

Thus for a sine wave the form factor is $\sqrt{2}=1.414$, whereas for peaky waves it may be as high as 2.

It is important to note that a peaked pressure wave will cause less core loss in transformers, motors, &c., than a flat-

topped wave for the same effective voltage, for with a peaked wave the effective or $\sqrt{\text{mean square}}$ value becomes greater in proportion than the mean value.

Since, with transformers also, the pressure developed by the secondary winding will be proportional to the primary current irregularities of the wave form applied to the primary terminals and accentuated by the capacity of the high-pressure feeder cables, will be reproduced upon the low-pressure voltage wave of the secondary circuits.

It is often of importance in the case of relays and protective gear to eliminate harmonics from the wave form of the secondary circuit of current transformers. This is usually effected by

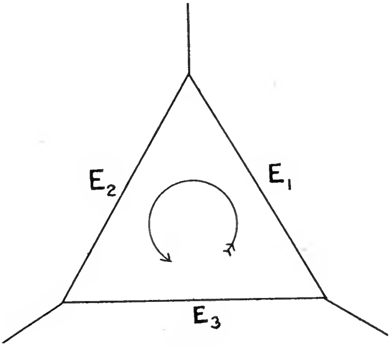


FIG. 53.

shunting the primary winding by a non-inductive resistance as shown in Fig. 52.

In this case the high frequency currents impressed upon the fundamental wave experience greater resistance to their flow through the inductive primary winding than through the non-inductive shunt, and in this way ripples in the secondary current wave tend to become smoothed out.

Rotation of Harmonics.

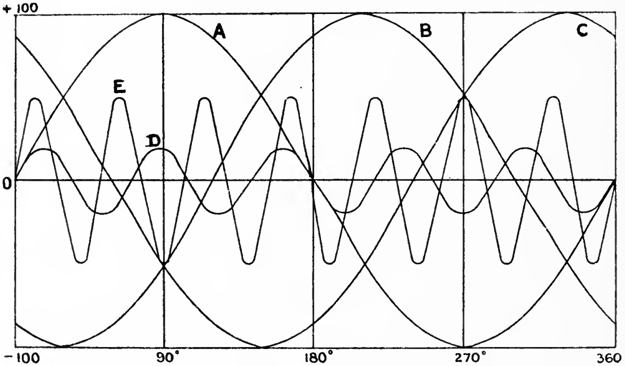
If three equal E.M.Fs., e_1, e_2, e_3, at phase differences of 120° be connected up in delta (Fig. 53), triple frequency harmonics will act in the same direction in all three phases at the same

time, and will produce circulating currents in the closed windings. The remaining harmonics differ in phase by 120° in each of the three windings.

Thus harmonics 1, 7, 13, 19, 25 combine at +120° phase, whilst harmonics 5, 11, 17, 23 combine at -120° phase. This is shown graphically by Fig. 54.

If the fundamental wave gives a rotating field in one direction, harmonics 7, 13, 19, 25 will give rotating fields in the

Rotation of Harmonics.



A, B, C = Phase Pressures.

A, B, C = $100 \sin \theta$, $100 \sin\left(\theta + \frac{2\pi}{3}\right)$, $100 \sin\left(\theta + \frac{4\pi}{3}\right)$.

D = 5th Harmonic = $20 \sin 5 . \theta$ (Retreating).

E = 7th Harmonic = $50 \sin 7 . \theta$ (Advancing).

Fig. 54.

same direction, whereas harmonics 5, 11, 17, 23 will give rotating fields in the opposite direction, the speed of field rotation being proportional to the frequency of the harmonic in each case. It will thus be evident that the presence of harmonics may have the effect of diminishing or increasing torque in induction motors and other plant.

Since only odd harmonics can exist in the wave form of commercial alternators, the relative frequency of these will be given by the expression $(2K-1)n$, where n is the frequency of

the fundamental wave and K has successive integral values from unity upwards.

The pressure wave of any alternator will, therefore, be represented by the sum of all such terms as :—

$$E_j \sin(2\pi jnt + \theta_j).$$

Where E_j is the amplitude of any harmonic.
j is the order of the harmonic.
n is the fundamental frequency.
θ_j is the phase displacement of the harmonic.

If the circuit to which this pressure wave is applied is of a non-inductive character and of resistance R, the current wave will be given by the sum of all such terms as :—

$$\frac{1}{R} E_j \sin(2\pi jnt + \theta_j).$$

If the circuit contains a constant inductance, the impedance offered to higher frequency current components will increase in proportion to their frequency, and these will tend to be smoothed out from the resulting current wave.

If the circuit includes a condenser on the other hand, the impedance offered to the higher frequency current components will be diminished, and accordingly these will be magnified in proportion to their frequency in the resulting current wave. Moreover, the phase displacement of the current components with regard to their corresponding pressure components will in each case vary with the reactance of the circuit.

Now for a sine wave the effective value or root mean square of its ordinates, the quantity indicated by a voltmeter is obtained by dividing the maximum ordinate by $\sqrt{2}$, hence with an irregular wave form the voltmeter reading will represent the square root of the sum of all such terms as—

$$\frac{E^2 j}{2}.$$

The instantaneous value of the watts is obtained by multiplying the instantaneous values of the volts and amperes, and is—

$$\frac{1}{R} (\Sigma E_j \sin(2\pi jnt + e_j))^2 \text{ for a non-inductive load.}$$

The average value of this in the case of an inductive circuit may be obtained by multiplying the ordinates taken from current

and voltage oscillograms, and then taking the mean. If capacity or self-induction be present, there will be a discrepancy in the values so obtained for reactive and non-reactive circuits respectively.

The impedance of a circuit of resistance R, self-induction L, and capacity K is—

$$\text{Impedance} = \sqrt{R^2 + \left(pL - \frac{1}{pK}\right)^2}$$

where pL is that part of the reactance due to self-induction and proportion to the frequency.

$\frac{1}{pK}$ that due to capacity and inversely proportional to the frequency.

Hence the impedance of the circuit to an harmonic of the nth order is—

$$\text{Imp}_n = \sqrt{R^2 + \left(npL + \frac{1}{npK}\right)^2}$$

It may be of interest to consider at this point the variation in wave form produced by long E.H.T. cables. We may

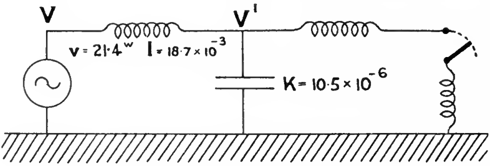


FIG. 55.

assume that a number of generators in parallel feeding the cable on open circuit have a combined self-induction which is negligible.

Take the case of a 20,000-volt 0.05 three-core cable 50 miles in length.

The Y capacity per core per mile is .211 microfarad, or for 50 miles 10.55 microfarads.

The self-induction per core per mile is .748 milli-henry, or for 50 miles 37.4 milli-henrys.

The ohmic resistance per core per mile is .858 ohm, or for 50 miles 42.94 ohms.

We may assume with close approximation that the total capacity is shunted across half of the line, as in Fig. 55.

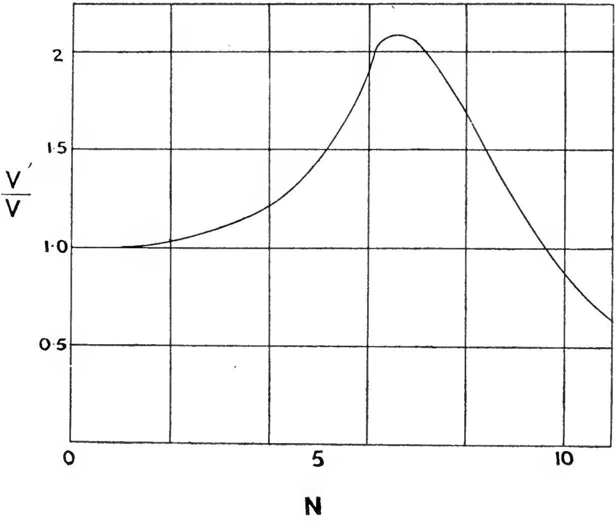


FIG. 56.

If the frequency n for the fundamental wave is 50 ~ and $p=2\pi n$; if V^1 = voltage at condenser, and V = voltage at generator, it may readily be shown that

$$\frac{V^1}{V}=\frac{1}{\sqrt{1-p^2(2lK-K^2r^2)+p^4(K^2l^2)}}.$$

Inserting values in the above expression and denoting by N odd multiples of the fundamental frequency, we get :—

$$\frac{V^1}{V}=\frac{1}{\sqrt{1-N^2(.034)+N^4(.000376)}}.$$

Taking values of N = 1, 3, 5, &c., in succession, we get corresponding values of $\frac{V^1}{V}$ as follows :—

n	$\frac{V^1}{V}$
1	1.016
3	1.10
5	1.435
7	2.06
9	1.18
11	0.64

It will be noted from the above ratios that the amplitude of the 5th harmonic is increased by over 43 per cent., whilst that of the 7th harmonic is increased by over 100 per cent. These results are shown graphically by the curve, Fig. 56.

CHAPTER VI

EARTHING

In the very early days of single-phase alternate current working with high potentials it was recognised that with a completely insulated system of generator and cables, the potential to earth of the system would oscillate about a certain value represented by some point, such as O in Fig. 57, this point being either at earth potential or some other potential above or below earth depending upon whether the system as a whole was electrostatically charged. In addition,

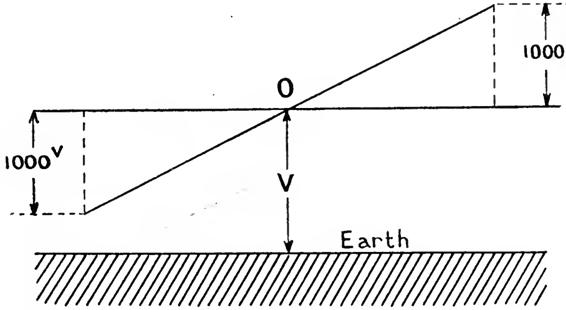


Fig. 57.

that the position of the point O would depend upon the leakage from the poles of the system to earth. It soon became apparent that cables which would prove sufficiently strong to work entirely insulated would, should their insulation become defective on one pole of the system, be subjected to a much greater pressure. For instance, if one pole of the generator became earthed, the potential in this case would oscillate about the point O′, Fig. 58, with double the amplitude of that in Fig. 57.

The practice of earthing one pole of the alternator permanently was then adopted, and the cable insulation made sufficiently strong to withstand the maximum variation in the potential of the system to earth likely to be met. This arrangement also proved of benefit in enabling measuring instruments, &c., to be inserted in proximity to the earthed terminal of the generator, and thus kept at earth potential.

A simple method for determining the potential of partially insulated systems has been developed by Mr M. B. Field. The following example will illustrate the principles involved.

In the diagram, Fig. 59, let OA, OB, and OC represent the

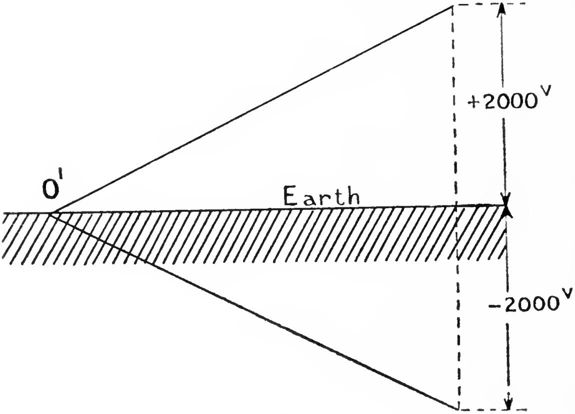


FIG. 58.

E.M.Fs. in the three windings of a star-wound generator. If the windings and circuits to which they are connected are completely insulated, it may happen that no part of the system is at earth potential. If the pole A be connected to earth the diagram may be considered as revolving round the point A as centre when the potentials of the poles C and B relatively to earth will be given by the lengths of the vectors AC and AB. If the three poles are all partially insulated, their relative potentials to earth will similarly be given by the revolution of the diagram about some other point, such as O′, in which case the line OO′ represents in amplitude and phase the potential

of the neutral point of the windings, and lines drawn from O′ to the three terminals A, B, C will similarly represent in amplitude and phase the potentials of these three terminals relatively to earth.

If, therefore, we can find the length and phase relationship of the line OO′ we know at once the potentials of each point of the system relatively to earth.

The length and position of the line OO′ is governed by the condition that the sum of all leakage currents flowing to earth is zero at every instant.

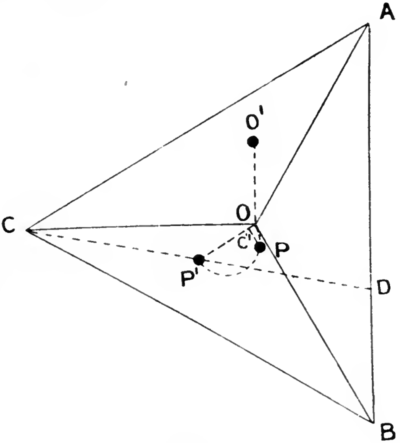


Fig. 59.

It may be readily shown that the position of the point O′ on the diagram is the same as the centre of gravity of three masses placed at A, B, and C, each proportional to the conductivity of the leakage paths to earth from these respective terminals.

As an example, assume the star-wound generator (Fig. 59) connected up to a cable system, the insulation of the cores in connection with each terminal being as follows:—

Terminal	A = 60,000 ohms	conductivity	$= 166 \times 10^{-6}$
„	B = 30,000 „	„	$= 334 \times 10^{-6}$
„	C = 20,000 „	„	$= 500 \times 10^{-6}$

The sum of the conductivities is thus $1{,}000 \times 10^{-6}$, or .0001, and

the total insulation of the system to earth is 10,000 ohms. Neglecting capacity currents in the first instance, the currents to earth from A and B alone would be equal if the line AB revolving around the point D, the lengths BD and AD being such that $BD \times 334 = AD \times 166$, or AD nearly twice BD.

Further, the total leakage paths from A and B of conductivity $\frac{166+334}{10^{-6}}$ or $\frac{500}{10^{-6}}$ may be considered as concentrated at the point D, and as the leakage path from C has an equal conductivity the point P′ midway along the line CD will represent the point at earth potential of the system, and the whole diagram may be considered as revolving round the point P′ as regards the relative potentials to earth of other points upon the system.

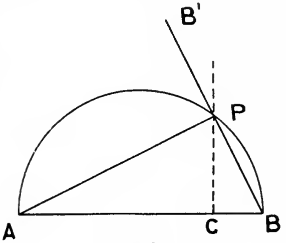


FIG. 60.

So far no account has been taken of the capacity currents flowing to earth from the three cores of the cable system in connection with the generator. These, as will be shown later, determine to a great extent the position of the point at earth potential on the diagram.

It will be necessary, however, to first consider the potential of a circuit which has leakage paths to earth due to conductivity and also to capacity.

Take the case of a single-phase generator whose E.M.F. is represented by the rotation of the vector AB, Fig. 60.

If we have paths of conductivity to earth in connection with each of the poles AB, the point at earth potential will be in some position between AB as already explained. Assume now that we have a capacity to earth of K farads in connection with terminal A, and a path to earth of resistance R ohms in connection with terminal B.

The current to earth through the capacity at A will be 90° in advance of the pressure inducing it, whereas the current to earth through the resistance at B will be in phase with the pressure inducing it.

The position of the zero potential point must be such that these two currents are equal and opposite at every instant.

It will be found that the locus of the zero point is a semicircle described upon the line AB as diameter.

Assume the diagram to rotate around the point P as centre, then the length of the vector AP will represent in phase and magnitude the pressure acting upon the capacity between terminal A and earth. This will produce a current 90° in advance of the pressure in the direction PB′, and of amount $2\pi n\mathrm{K} \times (\mathrm{AP})$ amperes. On the other hand the length of the vector PB represents in phase and magnitude the E.M.F. acting upon the resistance R, and inducing a current in it of amount $\frac{(\mathrm{BP})}{\mathrm{R}}$ amperes in the direction PB.

Now the condition that P is at earth potential is that these currents are equal and opposite, that is

$$\mathrm{AP} \times 2\pi n\mathrm{K} = \mathrm{BP} \times \frac{1}{\mathrm{R}}$$

or

$$\frac{\mathrm{AP}}{\mathrm{BP}} = \frac{1}{2\pi n\mathrm{KR}}.$$

If we drop a perpendicular PC from the point P we get the geometrical relation :—

$$\frac{\mathrm{AC}}{\mathrm{CB}} = \frac{\mathrm{AP}^2}{\mathrm{BP}^2} = \left(\frac{1}{2\pi n\mathrm{KR}}\right)^2$$

We may now proceed to consider the combined effect of capacity and resistance in the case of the star-wound generator, whose E.M.Fs. are shown diagrammatically by Fig. 59.

Assume that the generator is connected up to a one-mile length of 0.15 sq. in. three-core 5,000-volt cable. The Y capacity of this cable per core per mile may be taken at 0.285 microfarad, and if we assume a frequency of 50 cycles we get for the permittance of the leakage path

$$\frac{2\pi \times 50 \times .285}{10^6} = \frac{89.4}{10^6}.$$

As we have assumed that the capacity to earth in connection with each terminal of the generator is the same, which would usually be the case in practice, we may consider the total capacity leakage paths concentrated at the central point O of the diagram, and of permittance

$$\frac{89.4 \times 3}{10^6} = \frac{268.2}{10^6}.$$

The total conductivity leakage paths may similarly be considered as concentrated at the point P′ as before and of amount

$$\frac{1000}{10^6}.$$

Upon OP′ describe a semicircle, and divide OP′ at a point C′ such that

$$\frac{OC'}{C'P'}=\left(\frac{1}{2\pi n KR}\right)^2=\left(\frac{10^6}{2\pi\times 50\times 268.2\times 10000}\right)^2=\frac{1}{7.18}.$$

From the point C′ draw a perpendicular C′P to meet the semicircle at the point P. The point P will be the zero potential point on the diagram. It is important to note the considerable stability given to the neutral point of a star-wound generator by the presence of equal capacities of comparatively small amount between each phase and earth in the cable system to which it is connected. In practice there would usually be a much greater length than one mile of cable in connection with any generator running, and in such a case the zero potential point of the system would be indistinguishable from the neutral point of the generator, in spite of considerable variation in the ohmic resistance between each phase and earth.

The argument is sometimes put forward that with the neutral point of the system insulated there is less danger from shock to any one coming accidentally in contact with one pole of the system. Since, however, the effect of capacity is to keep the neutral point at earth potential, an attendant making contact between one pole and earth through his body would receive practically the same shock as if the neutral point of the generator be permanently connected to earth.

For the same reason electrostatic voltmeters connected between each pole of the system and earth will not indicate defective insulation until this becomes so low that a breakdown results immediately the voltmeters show an appreciable difference in their readings.

If the vectors OA, OB, and OC (Fig. 61) represent the E.M.Fs. of a star-wound generator by the rotation of the diagram about the neutral point O, and O*a*, O*b*, and O*c*, represent the amplitude of a triple harmonic, it is obvious that since the speed of rotation of O*a*, O*b*, and O*c* is three times that of A, B, C, whilst the line OA moves from the position OA into the

position OB, Oa, Ob, and Oc will have made one complete revolution and the E.M.F. Ob will now act on OA in its new position OB.

We see, therefore, that at the same instant the harmonic E.M.Fs. are acting from the neutral point O simultaneously in all three phases of the system.

Accordingly, if equal leakage paths exist in connection with the poles of a generator having its neutral point insulated, the potential of the neutral point will be raised above earth by the amount of the triple frequency E.M.Fs.

If the neutral point be not earthed, and the insulation of the system is perfect, the triple frequency E.M.Fs. will balance one another, and the pressure between the neutral point and each phase will not contain triple frequency ripples.

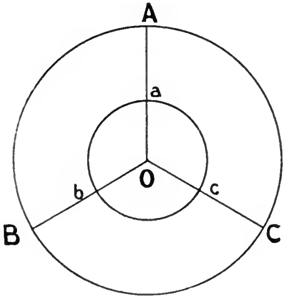


FIG. 61.

With the neutral point of the generator earthed, however, the triple frequency E.M.Fs. are superposed upon the pressure of each phase winding, and may increase or diminish the maximum value of the pressure wave according to the phase displacement of the triple frequency harmonics.

The pressure between phase and earthed neutral being expressed by V where

$$V = A \sin \theta + B \sin 3\theta + C \sin 9\theta, \text{ \&c.}$$

it has been shown in Chapter V. that the effective value of this or the amount indicated by a voltmeter is

$$V_1 \sqrt{\tfrac{1}{2}(A^2 + B^2 + C^2, \text{ \&c.})}.$$

Where three-phase star-wound alternators are run in parallel with earthed neutral points, some special conditions have sometimes to be met in practice. The usual connections are illustrated by Fig. 62, OO′ being the neutral points of the generator windings, AB a neutral bar earthed through the resistance R. The objects of earthing are threefold in the case of three-phase generators feeding transmission circuits: (1) To maintain the correct relative potential between each

phase and earth; (2) To restrict the current through a fault on the cable system to that sufficient to operate the circuit breakers clearing the fault; (3) To dissipate electrostatic charges produced by external or internal influences.

It will be evident that if the wave form of one machine differs from that of the others in possessing pronounced triple frequency harmonics, heavy currents may circulate within the closed circuits formed by the windings of the machines in parallel. Moreover, the direction of these triple frequency

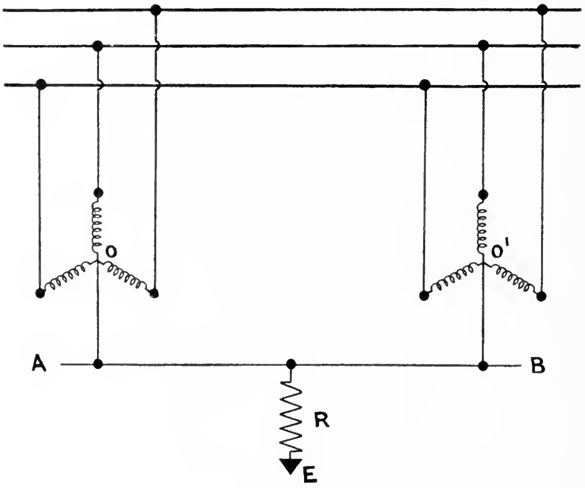


Fig. 62.

currents in the phase windings being such as to produce opposite magnetising effects upon the stator iron, the parallel circuit of three branches between each neutral point will tend to be non-inductive.

Various devices have been adopted and proposed for preventing these triple frequency circulating currents. In some cases the neutral point of one machine alone is earthed.

The use of choking coils and a combination of choking coils and condensers has also been suggested, placed between the neutral points of the generators and the earthed bar. It is

evident that the impedance to triple frequency currents offered by a choking coil possessing approximately constant self-induction would be nearly three times that opposed to currents of fundamental frequency, and hence the circulating currents would be damped out to a greater extent than currents of normal frequency to earth due to a short circuit on the cable system.

The combination of a suitably proportioned capacity and self-induction in the earth circuit is also interesting, since this combination can be tuned to offer practically no impedance to currents of normal frequency, whilst possessing considerable impedance for currents of triple frequency.

A further method of eliminating these circulating currents

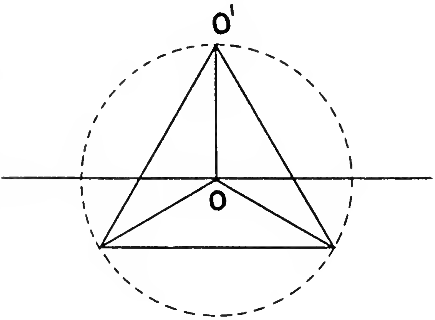


FIG. 63.

consists in the employment of three single-phase transformers having a ratio of 3 to 1, the secondary windings being connected in series between the neutral point of the generator with irregular wave form and the earthed bar. The resultant triple frequency E.M.F. is thus made to cancel that existing between the neutral point of the generator and earth.

Three delta-connected transformers are frequently employed for feeding a transmission line; under normal conditions, with equal leakage currents from each line and assuming the absence of electrostatic charges from lightning, &c., we may consider the potential of the transformer windings relatively to earth, as given by the rotation of an equilateral triangle about its centre of gravity O, as in Fig. 63.

If from any reason, however, the leakage current from any one line is much in excess of that from the other two, the axis of rotation of the triangle will be transferred from the point O to a point near the pole O′, and the whole triangle will rotate about this point (Fig. 64). In this case it will be seen that the high-pressure windings of the transformers will be charged alternately plus and minus to the full potential between line wires.

Now, the low-pressure winding of the transformers and ad-

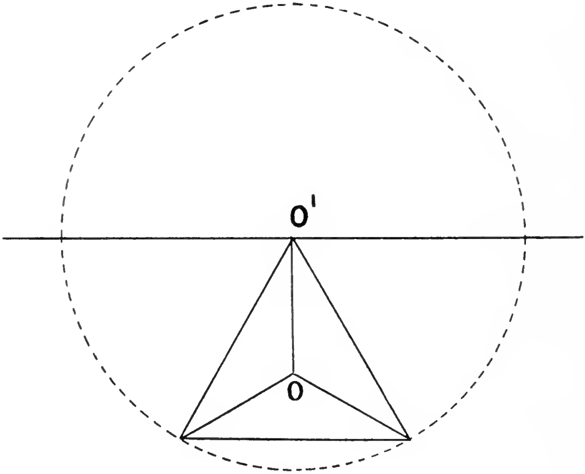


Fig. 64.

jacent high-pressure windings will possess considerable electrostatic capacity. Moreover, if the generator feeding the low-pressure windings is entirely insulated from earth, an additional capacity will exist between the generator windings and primary windings of the transformers and earth. We shall, accordingly, have the condition of things represented by Fig. 65.

The total line pressure will be divided between the two condensers A and B in series in the inverse ratio to the capacities.

As an example, assume a 5,000-volt generator having a capacity between windings and earth of .003 microfarad per

phase, whilst the capacity between the windings of each transformer is .001 microfarad. It is evident that with a line pressure of 60,000 volts the generator insulation may be subjected to a pressure of 15,000 volts, whilst the insulation between the windings of the transformers will be subjected to a pressure of 45,000 volts.

A pressure of three times the working pressure would be very severe upon the generator insulation and likely to cause breakdown, whereas by the simple expedient of earthing the neutral point this danger would be averted.

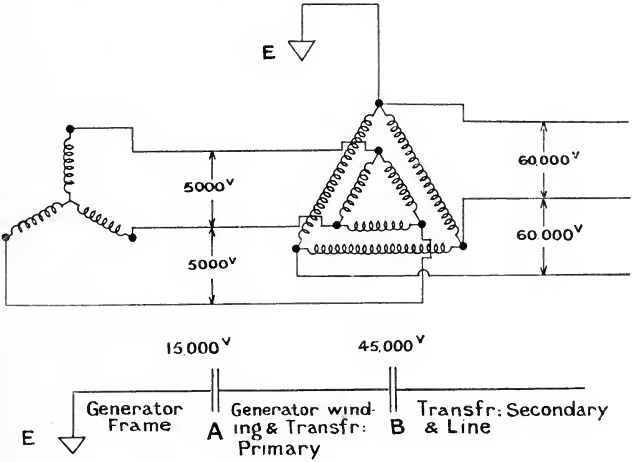


FIG. 65.

Similar considerations will exist as regards the receiving end of the line where step down transformers are employed.

With transformers wound for very high potentials such as 100,000 to 200,000 volts, the windings are found to possess considerable capacity, and, in addition, more or less self-induction due to magnetic leakage.

When one pole of the high tension winding of such a transformer is earthed, and the other pole is on open circuit, the capacity current will be a maximum at the point where the windings join the earth connection.

If, on the other hand, both poles of the high tension winding are insulated and the mid-point of this winding is available, it will be found that the maximum value of the capacity current is at the mid-point of the winding.

The capacity currents in the E.H.T. secondary windings of such transformers on open circuit react upon the primary magnetising current, and in some cases may render this a leading current instead of a lagging one.

It may thus happen that resonance occurs due to the capacity and self-induction of the H.T. windings causing very considerable and sudden rises of pressure.

A type of extra high-pressure transformer designed by the Oerlikon Company is illustrated by Fig. 66. The transformer is enclosed in an oil tank, and pipe coils for circulating cooling water are arranged in the upper layers of oil. This type of transformer is used for normal working pressures of 40,000 volts and upwards.

Where long transmission lines are used to convey power for lighting and industrial purposes to densely populated towns, it will generally be necessary to terminate the overhead lines at a substation outside the town, the supply being then transformed to lower pressure and distributed to other substations within the town itself by an underground network of cables. In such cases it is of the utmost importance to protect the underground cables at the point where they are connected to the overhead conductors through the transformers. In these cases the principle of earthing through a resonant circuit has also been effected with success. The substation building containing the transformers, switchgear, &c., is made to form a metallic cage by means of wire netting enclosed in the walls of the building and effectively earthed throughout. Each of the transmission lines is then connected through a resonant circuit consisting of a condenser and self-induction in series to the earthed metal cage. The resonant circuits are tuned to some value between the usual frequencies met with due to lightning discharges or high frequency power surges, and of the order of 50,000 to one million per second, to which they will offer little impedance. For the ordinary working frequency of the transmission line, however, the impedance of these resonant earth circuits will be almost infinite. In this way complete protection to the cable system has been secured.

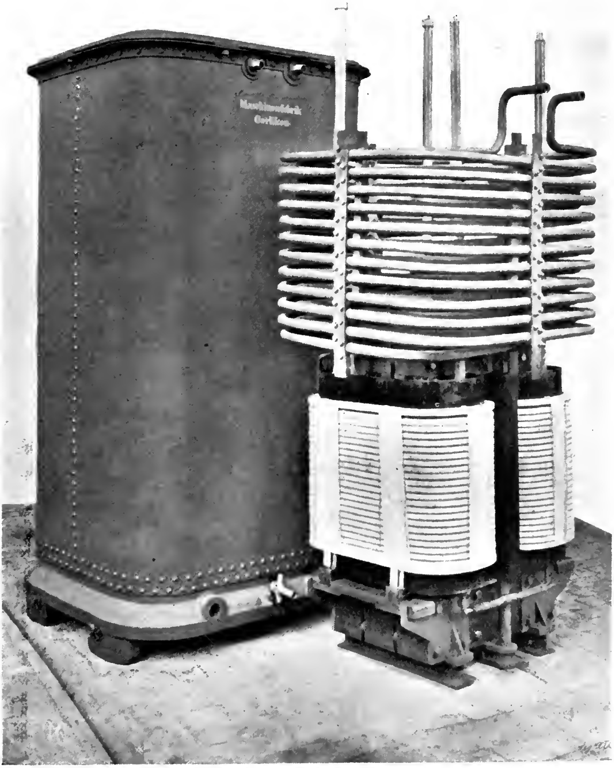


FIG. 66.

In cases where the distribution of power to consumers is carried out by three-phase four-core cables, complete protection of the low-pressure distributing mains from abnormal rise in pressure, due to leakage from the E.H.T. circuits of the system, is obtained by joining up the secondary windings of the transformers in star with the neutral point of transformers, and the fourth neutral conductor of each distributing main connected to an earthed bar at the substation. Where the low-pressure distribution, however, is also carried out by means of overhead conductors or three-core cables, additional safety devices may be required to safeguard against leakage from the extra high-pressure portions of the system. Such devices usually take the form of star-connected electro-magnets with shaded poles operating relay discs. The neutral point is earthed and the other end of each magnet winding connected to one of the three phases of the distributing mains. Any abnormal rise in pressure causing a disturbance of the relative potential between the low-pressure conductors and earth will thus operate the relays and cut off the low-pressure supply.

CHAPTER VII

LINE APPLIANCES

Supports.—Line supports are required to resist stresses under ordinary conditions due to the following:—

(1) Weight of wires assumed to act vertically.
(2) Wind pressure on wires assumed to act horizontally.
(3) Wind pressure on pole and brackets and insulators.

If the direction of the line changes abruptly the resultant stress upon the support due to the tension in the wires is usually counteracted by the employment of extra stays or stronger supports.

Occasionally the supports will be subject to unbalanced stresses due to broken wires, but under normal conditions stresses due to wind pressure on the supports and wires are the chief considerations. For long spans, lattice-work iron towers (Fig. 67) are now coming into general use, and possess several advantages.

In designing an overhead line for power transmission at high pressure, the Board of Trade require that a maximum wind pressure of 30 lbs. per square foot be allowed for acting normally to any flat surface. In the case of cylindrical surfaces, the wind pressure is found to be only about one-half to two-thirds of that on a flat surface. In calculating the effect of wind pressure, therefore, it is usual to take half of the projected surface in the case of wooden poles and two-thirds of the projected surface in the case of the wires.

If R = mean radius of a pole in inches.
L = length of pole out of ground in inches.
L_1 = height from ground to centre of wires in inches.

$$\text{The wind pressure at 30 lbs. per sq. ft.} = \frac{RL}{4.8} \text{ lbs.}$$

As the resultant pressure acts at a point approximately half

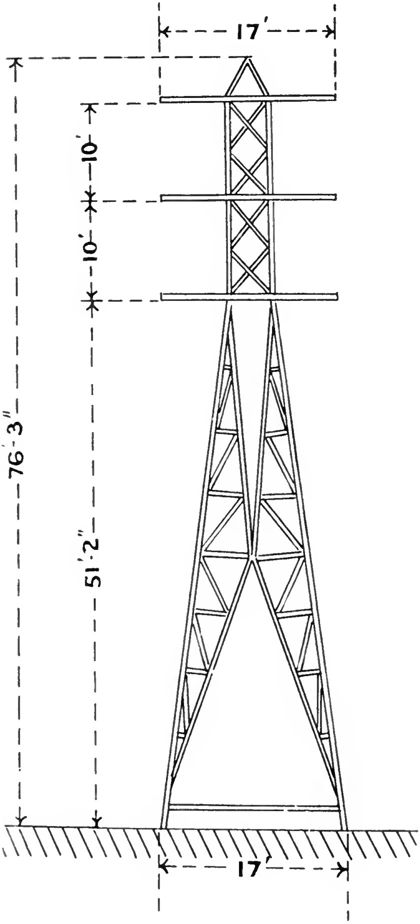


FIG. 67.

the height of the pole, the equivalent pressure acting at the centre of the wires is :—

$$\frac{RL}{4.8} \times \frac{L}{2L_1} = \frac{RL^2}{9.6\ L_1}.$$

Similarly, it may be shown that the wind pressure on a single wire is 1.67 dS taking two-thirds of its cylindrical diameter,

where d = diameter of wire in inches.
S = length of span in inches.

The Board of Trade Regulations require that the maximum stress in overhead wires at a temperature of 22° Fahr. and with a maximum wind pressure of 30 lbs. per square foot shall not exceed one-fifth of the breaking stress of the wire. It is usual to assume that the curve taken by a suspended wire is a parabola, when the tension due to the weight of the wire alone is given by :—

$$\frac{W \,.\, l^2}{8 \,.\, S};$$

where W = weight in lbs. per foot.
l = length of span in feet.
S = sag of wire in feet.

This formula also holds for any other consistent units of length employed.

For copper the tension in pounds is given approximately by :—

$$\frac{5.8 \times \text{area square inch.} \times (\text{span in feet})^2}{\text{Sag in inches}}.$$

As there will be a certain amount of elastic extension of the wire, the sag will be more than given by this formula. The testing of the elasticity of any given wire is readily carried out, however, in practice by observation of the sag over different lengths of span.

The tension due to wind pressure of 30 lbs. per square foot with a suspended wire of diameter d, all lengths being taken in the same units, is :—

$$\frac{l^2}{8s} \times \left(\frac{d}{8}\right),$$

and the tension due to the resultant of weight of wires and wind pressure is :—

$$T = \frac{l^2}{8s}\sqrt{W^2 + \left(\frac{d}{8}\right)^2}.$$

It is obvious from the above formula that smaller wires experience greater stresses proportionally than those of larger diameters.

The correct design of metal lattice work and other types of supports for overhead transmission lines involves a number of considerations common to all engineering structures, and fully dealt with by purely mechanical treatises. It is only proposed to consider here briefly some of the simplest types of supports, and to indicate a few of the general principles underlying their construction.

It is shown in books on mechanics that if a bending moment equivalent to a weight W suspended at the end of an arm of length l acts at any section of a beam having moment of inertia I, the material will be stressed to the extent $\frac{Wl}{I}$ at 1 in. from the neutral line of the section. If the edge of the section is n inches away from the neutral line, the maximum stress will be n times this amount, and should this stress exceed the breaking stress of the material fracture will occur.

The quantity $\frac{I}{n}$ is generally called the strength modulus of the section.

For rectangular sections this is given by—

$$\frac{bd^2}{6},$$

where b = breadth and d = depth.

For a circular section, the strength modulus is—

$$\frac{\pi}{4}R^3,$$

where R is the radius of the section.

If we denote by F the breaking stress of the material in pounds per square inch, the bending moment M at fracture for a rectangular support fixed at one end and loaded at the other is—

$$M = F\frac{bd^2}{6},$$

and for a support of circular section it is—

$$M = F\frac{\pi}{4}R^3.$$

The value of F having once been obtained from a destructive test of one support, the bending moment by which any similar support will be ruptured may be calculated.

From a number of tests of fir poles it was found that the load in pounds W which, applied at a height of L_1 inches above ground level to a pole of radius R inches at ground level, would produce fracture, could be expressed by the following formula :—

$$W = 6128\frac{R^3}{L_1}.$$

We have also seen that the equivalent wind pressure w on a pole acting at a height of L_1 inches from this ground level was

$$w = \frac{RL^2}{9.6\ L_1}\ \text{lbs.}$$

Hence, with a factor of safety of ten, the net strength which is available for resisting lateral wind pressure on the wires is

$$\left(\frac{W}{10} - w\right)\ \text{lbs.} = \left(612.8\frac{R^3}{L_1} - \frac{RL^2}{9.6\ L_1}\right)\ \text{lbs.}$$

It is desirable that the taper of poles is such that the radius at the top is not less than two-thirds of the radius at ground level to ensure that the greatest strength occurs at this point, which is most subject to decay.

The deflection of a support fixed at one end and loaded at the other is given by

$$D = \frac{WL_1^3}{3EI},$$

where E = Young's Modulus.
I = moment of inertia of the section.

The value of I for a circular section is

$$= \frac{\pi R^4}{4}.$$

Hence we may write :—

$$D = \left(\frac{4}{3\pi E}\right)\frac{WL_1^3}{R^4}.$$

The deflection of a standard support having once been ascertained by test and the value of the constant term in brackets determined, the deflection of any support of similar type and material can then be calculated.

Great stresses sometimes occur due to the breaking of all the line wires in one span, resulting in a heavy pull on the supports by the tension of the wires in those spans near the break which are not interrupted. If a support is flexible in the direction of the line, the tension in the wires will be much reduced by the bending of the supports, and probably by the foundations giving also to some extent. The support will deflect in such cases until the tension of the wires is equal to the elastic resistance of the support.

From a number of tests of fir poles, the deflection (D) was found to be approximately :—

$$D = \frac{1}{3456000} \frac{WL^3}{R^4}.$$

A Poles.—These (Fig. 68) should have an 8 in. by 4 in. creosoted brace block 6 to 8 ft. in length fixed about 2 ft. from the butt end, and at the top the two legs should be scarfed together, about one-third of each pole being cut away. An oak key 6 in. deep let into each member 1 in. to $1\frac{1}{2}$ in. about two-thirds from the top of the scarf prevents slipping.

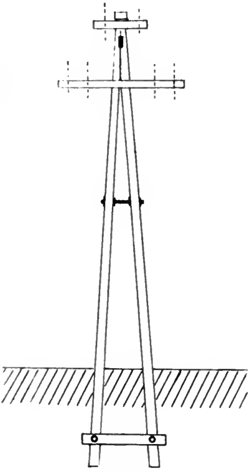


Fig. 68.

The spread of A poles should be about one-eighth of the length.

An A pole is four and a half times as strong as a single pole similar to those composing its members.

The wind pressure on an A pole is taken as one and a half times that on a single pole.

The deflection in the direction of the wires is only about half that of a single pole.

Taking average values of tests by Professor Goodman, it appears that the transverse deflection of an A pole is about one-fiftieth that of a single pole of the same dimensions as those composing its members. Considerable variation is likely to occur, however, owing to slip at the

joint. The majority of poles tested to destruction failed through the buckling of the compression leg.

Cross Arms.—The length of cross arm depends upon a number of considerations in the case of long high voltage transmission lines. These have already been referred to in Chapter I.

On short medium pressure lines the possibility of short circuit due to the swaying of the wires in the wind becomes of importance. Birds are also likely to cause short circuit on such lines.

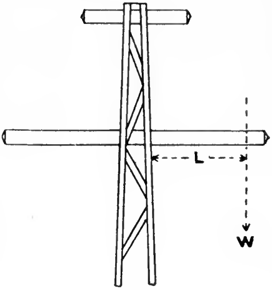


FIG. 69.

The Breaking Stress W *of a Rectangular Oak Cross Arm* (Fig. 69) is given by

$$W = \frac{CBD^2}{L},$$

where W = weight in cwts. at the extremity of the cross arms,
L = length of arm in inches.
B = breadth in inches.
D = depth in inches.
C = A constant = 17 approximately.

Economical Span.—The total cost of overhead lines is made up of:—

Cost of wires.
„ supports.
„ insulators.
„ erection.
„ wayleaves capitalised.

In the case of any particular overhead line the cost of the supports will be proportional to the number employed. Wayleaves will also largely depend upon the number of supports adopted.

If, however, the number of supports is decreased, they must be made stronger and higher to resist increased wind pressure, and to give sufficient headroom. The cost of labour during erection will also be increased per support.

It is, therefore, apparent that there is a most economical span which can be adopted with any given set of electrical requirements.

It is found that over a fair range of spans, however, the total cost is fairly constant.

The stress on supports due to change in the direction of the wires may be ascertained as follows. If the wires make an angle ϕ at the support B (Fig. 70), the total tension in directions BA and BC will be nt, where n = the number of wires, and t = the

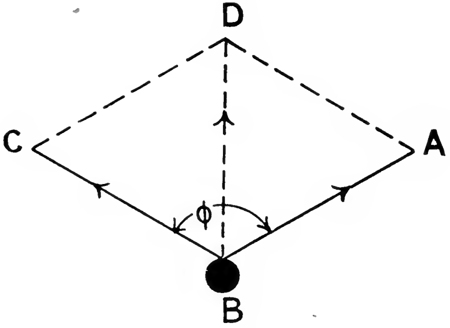


FIG. 70.

tension of each wire in lbs. The resultant T of these two forces in the direction BD is

$$T = 2nt \cos \frac{\phi}{2} \text{ lbs.}$$

The stress in practice would usually be taken up by a galvanised steel stay wire consisting of seven or nineteen strands of No. 8 S.W.G. wire.

If in Fig. 71 T = resultant stress in line wires as before,
S = stress in stay wire,
ϕ = the angle the stay wire makes with the support,

then
$$S = \frac{T}{\sin \phi} \text{ lbs.}$$

If the stay wire cannot be fixed to the support at the same level as the line wires (Fig. 72)—

Let $\quad H$ = height of line wires above ground.
h = height at which stay wire is fixed.

Then
$$S = \frac{T}{\sin \phi} \cdot \frac{H}{h}.$$

It is to be hoped that much greater facilities than hitherto will be extended to promoters of transmission schemes in this country in the near future.

The Board of Trade Regulations requiring high factors of safety largely increase the cost of construction of overhead lines, and the difficulties of obtaining wayleaves often militate seriously against an economical transmission of power into rural districts.

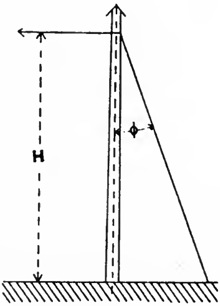


FIG. 71.

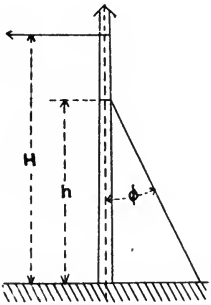


FIG. 72.

A further obstruction sometimes arises from the powers conferred upon Local Authorities to veto overhead wires, and some amendments of these legal powers of obstruction are sorely needed.

Telephones.—With all extra high-pressure transmission and distribution schemes in this country the Board of Trade insist upon adequate telephonic communication being provided between the generating stations, distributing stations, and substations of the system respectively. This is usually carried out by multi-core paper and air-insulated telephone cables, lead-covered and armoured, laid direct in the ground. These telephone cables are in most cases of necessity laid alongside the high-pressure feeder cables traversing the same route, and a certain amount of

interference is sometimes found to occur between the telephone and high-pressure circuit. In the writer's experience, where the neutral points of the generators at the power station are earthed, such interference in some cases has been traced to the insufficiently insulating qualities of the small wires run between the telephone cable dividing boxes and the instruments themselves. It must be remembered that with irregular wave forms containing triple frequency harmonics the lead sheaths of the feeder cables are subject to electrostatic charges of high frequency, and in spite of the usual earthing arrangements small sparks may often be obtained between the cable sheaths and neighbouring metallic objects. With overhead transmission lines working at pressures of 100,000 volts the telephone lines are usually run upon an independent set of poles.

With lines working at lower pressures, however, it is common practice to run the telephone wires at about 10 feet below the line conductors attached to insulators carried by the line supports. Electro-magnetic induction is largely prevented by the transposition of both the line conductors and the telephone conductors themselves.

Where the line transformers are arranged in delta connection, however, a further precaution against electrostatic effects is sometimes adopted. This consists in joining the telephone wires by split choking coils at each end of the line, the mid points of these choking coils being connected to earth.

Lightning Arresters.—The types of lightning arresters in most general use for protecting transmission lines may be described briefly under the following headings:—

1. Electrolytic.
2. Water jet.
3. Multigap.
4. Horn gap.

The electrolytic form of lightning arrester which is now very extensively used is based upon the principle that aluminium immersed in a suitable electrolyte becomes coated with a film which will only allow a small current to pass until a certain voltage is reached, approximately 400. After this voltage is exceeded the film breaks down, allowing a large current to pass, but upon the reduction of the voltage the film is again reformed.

A large number of circular dished shaped trays of aluminium are fitted one within the other, but separated by suitable insulated washers.

The space between the trays is filled with electrolyte, and the whole enclosed in an earthenware jar. For high voltages a number of such jars are mounted one above the other, and connected in series with a horn gap between each line and earth.

With the water-jet type of arrester water is allowed to spray upwards and impinge upon horizontal metallic plates in direct connection with the line wires. It is generally only employed at the generating station end of the transmission line or at substations where a plentiful supply of water is available. Although this type of arrester has been found to work well on the Continent it entails a loss of power by reason of leakage from the transmission line, and for this reason it would appear to be the usual practice to shut off the water unless thunderstorms are expected.

In the case of the multigap arrester a number of small metal cylinders are employed, separated by small air gaps, the number of gaps being made sufficient to prevent arcing after a discharge from line to earth has taken place. In some forms of this arrester about twice the number of gaps required to resist the line voltage are employed in series with a resistance to earth. In addition a shunt resistance is placed across the half of the air gaps remote from the line.

The horn type of arrester (Fig. 73) consists of wires placed in a vertical plane at an angle between them, the air gap being least at the bottom, but increasing upwards. One of the wires is connected to the line, and the other to earth usually through a water resistance. The space between the horns at the bottom is adjusted to withstand a discharge at a certain margin over the normal pressure between line and earth.

Generally speaking, in the case of long transmission lines lightning arresters are only installed at the ends of the line, or at substations. It is now becoming the usual practice, however, to run an earthed wire the whole length of the transmission line situated at some distance above the line conductors, and earthed at frequent intervals.

Choke Coils.—An important accessory to every overhead transmission line is the choke coil or self-induction spiral.

Fig 73

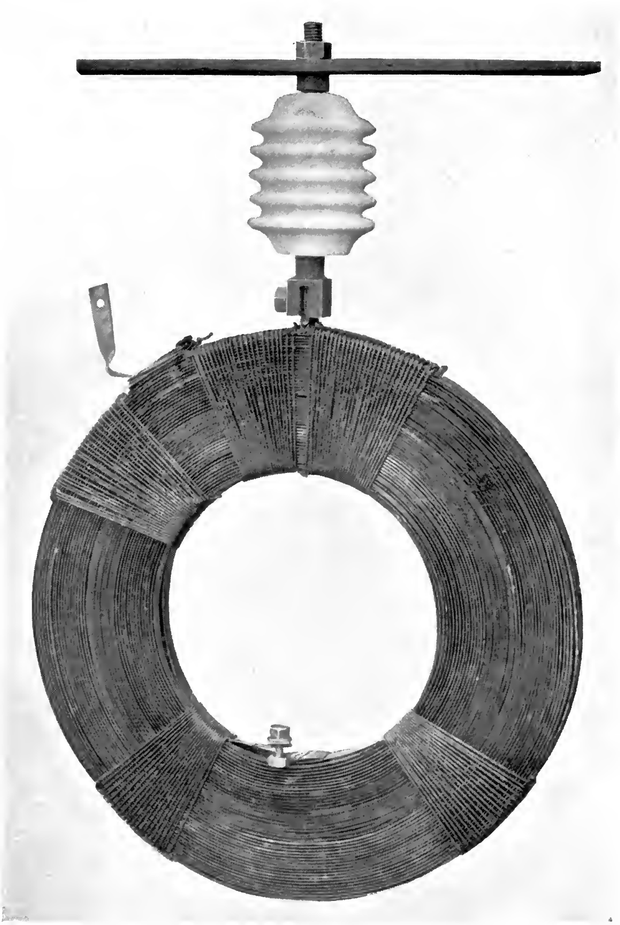


FIG. 74.

Each line after passing the lightning arresters at the entrance to the generating station or substation passes through a choke coil (Fig. 74), generally consisting of two flat spiral coils of copper placed side by side, and connected in parallel. These coils are usually supported in mid air by porcelain insulators, but in some cases are immersed in oil. The special feature of lightning and other static disturbances on transmission lines is the extreme rapidity with which they occur, and their very high frequency of oscillation. It has been computed that the frequency of a lightning discharge may be several million periods per second, and probably is seldom less than 50,000 periods. Accordingly choke coils which at the normal frequency of 25 to 60 periods of the supply would be practically non-inductive, behave as insulators to impulsive rushes of current, and rapidly oscillating discharges. Hence by placing suitable choke coils on each line upon entering the generating station, and after the line has passed the lightning arresters, most static disturbances will be dissipated through the lightning arresters. Choke coils are shown in position in Fig. 12.

Power Factor Correction.—The power factor of a circuit is usually defined by the ratio of true watts, as measured by a wattmeter, to the apparent watts obtained by the product of the effective or $\sqrt{\text{mean}^2}$ amperes flowing in the circuit by the potential difference in effective volts across its terminals. With sine waves of pressure and current the power factor is also given by the value of the cosine of the angle of phase difference between the pressure and current waves respectively. The power factor can never be greater than unity, and in the case of most alternating-current circuits met with in practice is less than unity.

With irregular wave forms of pressure and current the power factor of the circuit will be dependent upon other characteristics, these being the angles of phase difference between corresponding harmonics present in the pressure and current waves and variation in proportionality between such harmonics. It is, therefore, evident that the phase difference between an irregular pressure and current wave as determined by the points at which they cut the horizontal axis respectively, cannot be used to determine the power factor of the circuit, since the positions of these zero points may be altered considerably by

the presence of certain harmonics, as we have already seen in Chapter V.

Generally speaking, if we find the fundamental sine components of the pressure and current waves by harmonic analysis, we shall find also that the phase difference between them largely determines the value of the power factor of the circuit, the effect of other harmonics often being inappreciable.

In all cases where the power factor of a circuit is low, due to phase difference between pressure and current, it is possible to raise it by placing a suitable condenser or self-induction as a shunt across the circuit, but in special cases, where low power factor is due to wave distortion, this method cannot be adopted.

The two principal causes of low power factor in practice are :—

(1) Self-induction of load and line.

(2) Capacity of line.

The extent to which the regulation of an overhead transmission line may be affected by these quantities has already been discussed, and it is now proposed to review briefly the practical means available to raise the power factor of a circuit.

The effect of low power factor upon extensive systems employing expensive underground mains and many substations may be very serious, and accordingly may justify considerable capital expenditure towards rectifying the defect.

For instance, the steam plant in the generating station may be prevented from working up to its full rated capacity, due to the fact that the output of the alternators is limited by the heating of their windings, and the demagnetisation of their field magnets by the large lagging currents.

Moreover, the inductive drop in the cable system with low power factor will greatly exceed that at unity power factor.

The C^2R loss in the cables will indeed vary inversely as the square of the power factor, thus with a power factor of 0.5 the C^2R loss will be four times that with power factor unity.

As regards the substations themselves the regulation of the transformers may be seriously affected by low power factor and the capacity of both transformers and switchgear heavily handicapped.

Where the low power factor of the system is due to a highly inductive load such as that formed by a number of induction motors working considerably under full load at the end of the

line, the generators, line, &c., may be relieved of the wattless component of the current by shunting the load by a suitable condenser, or applying its equivalent in the form of a synchronous motor over-excited, or by the employment of rotary converters. This arrangement is represented diagrammatically

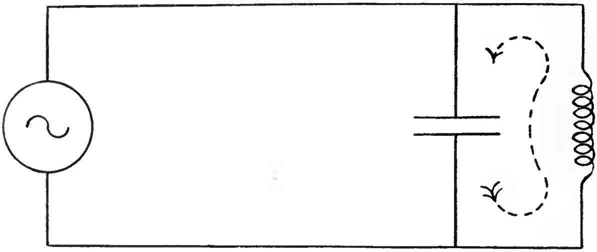


FIG. 75.

by Fig. 75; the wattless component of the current in this case only circulates locally between the load and the shunt formed by the condenser or its equivalent.

On the other hand, if the low power factor is due to the electrostatic capacity of a long overhead transmission line at

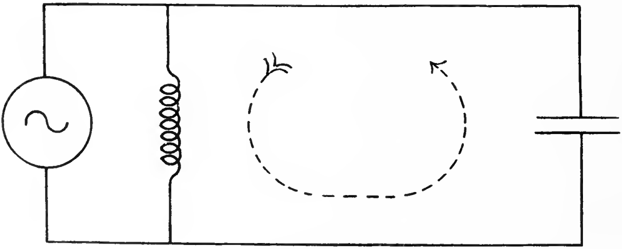


FIG. 76.

light loads, the generators alone can be relieved of the wattless component of the current by shunting the line with a suitable self-induction as shown diagrammatically by Fig. 76.

In reviewing the practical means available for correcting low power factor upon an existing system, it is first to be noted that the extent of the correction required will vary from time to time

with the load throughout the day. Accordingly, in those cases where induction motors are the cause of the trouble, and the employment of condensers is adopted, it will probably be found most convenient to arrange that each of the larger units be provided with its own condenser which can be switched on to the supply circuit simultaneously. If synchronous motors or rotary converters be used, however, a certain amount of hand regulation will generally have to be faced. The manufacture of condensers suitable for extra high-pressure working has

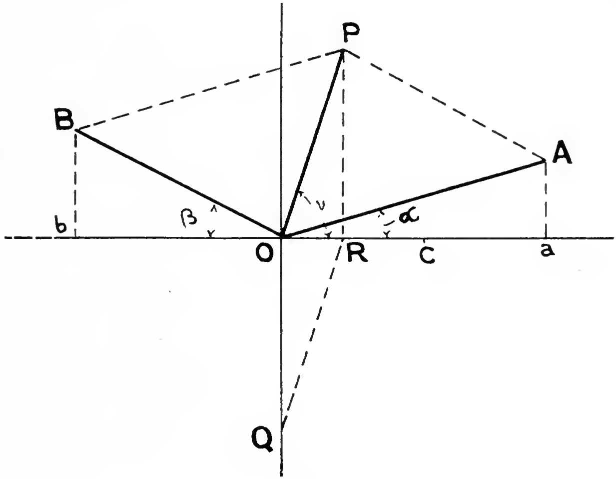


FIG. 77.

already attained considerable success, the principal factors militating against their more extended use being high initial cost and the difficulty of dissipating the heat produced in the dielectric under working conditions.

For correction of low power factor due to the electrostatic capacity of long transmission lines, large inductances have been successfully employed to relieve the generating plant in some cases of thousands of apparent kilowatts represented by the large charging current flowing into the line at the high working pressure adopted.

With an existing load of induction motors at the end of a transmission line, resulting in a low power factor of the system, the most efficient means of correction will usually be found in the adoption of over-excited synchronous motors "floated" or working on the line at the receiving end, and in close proximity to the load. It may, therefore, be of interest to consider here some features upon which the success from a financial point of view of this method of power factor correction will depend.

In Fig. 77,

Let OA = E.M.F. of generator.
OB = back E.M.F. of motor.
OC = current in circuit.
OQ = PR = back E.M.F. of self-induction of circuit = pLC.
OR = CR loss in circuit.

Then OP = E.M.F. required to drive the current C round the circuit, and the phase difference between OA and OB must be such as to give OP as a resultant.

From the figure it is evident that the output of the generator is OA × OC × cos α watts, whilst the power used by the motor is OB × OC × cos β watts in furnishing power and overcoming friction losses, &c. Since OB and PA are equal and parallel it is evident that their projections Ob and Ra are equal, and, therefore, the difference between Oa and Ob is OR, and OC × OR is the power lost in resistance.

In Fig. 78 let the resultant pressure OP have the same value and phase as before, and hence the current C remain the same, but let the counter E.M.F. of the motor OB′ be greatly increased, keeping the point B′ in the same vertical line bB, so that the power used by the motor OB′ × OC cos β' will remain the same as before. In this case the phase position of the impressed E.M.F. OA′ required to complete the parallelogram will lie on the reverse side of the axis OA to that in the case of Fig. 77, that is, the current OC will now be in advance or lead the impressed pressure OA′.

We see, therefore, that by over-exciting a synchronous motor so that the counter E.M.F. exceeds the impressed E.M.F., the current in the circuit will lead the impressed E.M.F.

From the same diagram it may be deduced that there are two values of the counter E.M.F. or excitation for which the

current in the circuit will be the same, lagging or leading the same impressed pressure by equal angles. Moreover, for a certain value of counter E.M.F. or excitation the current in the circuit will be a minimum.

As an example of power factor correction by means of a synchronous motor we may take the case of a generating station with an output of 4,000 kilovolt-amperes at a total power factor of 0.7, and assume that it is desired to raise this power factor to 0.9.

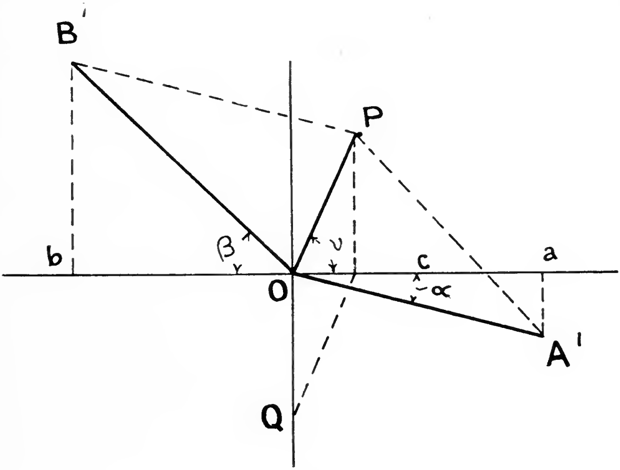


FIG. 78.

Take any horizontal line OX (Fig. 79) to represent the phase of the pressure at the generating station, and with radius OA = 4,000 units describe an arc of a circle AB. Upon OX mark off a length OC = 2,800 units, and from C draw CD at right angles to OX, cutting the arc AB at the point D. The length OC will now represent the true power in the system, 2,800 kilowatts. CD will represent the wattless component of the power, 2,860 kilovolt-amperes, and the length OD the total output of the station in kilovolt-amperes at 0.7 power factor.

Upon OX mark off a length OE = 3,600 units, and from th

point E draw EF at right angles to OX, cutting the arc AB at the point F.

The length OE will now represent true power in the system to the extent of 3,600 kw., EF will represent the wattless component of the power 1,746 kilovolt-amperes, and OF will again represent the total output of the station of 4,000 kilovolt-amperes at 0.9 power factor. Now, in order that the power

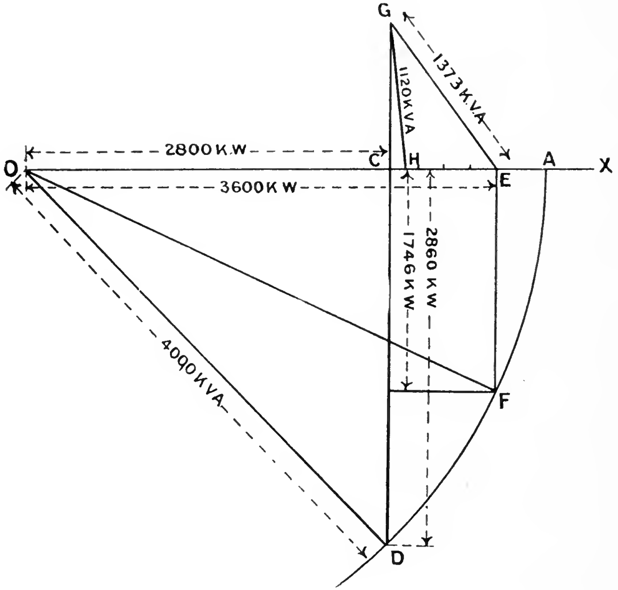


Fig. 79.

factor of the system may be raised from 0.7 to 0.9, it is obvious from the diagram that the difference between the lagging components CD and EF, or 1,114 kilovolt-amperes, must be counteracted by a leading component of like amount furnished by the synchronous motor. From the point C erect a perpendicular CG on the line OX of length 1,114 units, this will represent the leading wattless component to be furnished by the synchronous motor. We now require to estimate the real power required to

overcome friction, windage, iron and copper losses in the motor running light. Assume this to be 100 kw. and mark off CH of length 100 units in phase with the pressure OX. CH then represents real power lost in the motor. Join GH, which will be found on measurement to be approximately 1,120 units and gives the kilovolt-ampere rating of the synchronous motor, which, floated on the line, will raise the power factor of the system from 0.7 to 0.9.

The first thing to be noted in connection with the above calculation is that although the useful capacity of the generating plant has been increased by 800 kw., this has been effected by the necessary capital outlay upon an 1,120 K.V.A. synchronous motor, and in addition the running cost of 100 units of electrical energy per hour. This brings us to the all-governing question with any scheme for power factor correction, "Will it pay?" As already indicated, the answer will depend in any particular case upon the capital cost of plant mains and substations running charges, &c., which expert investigation will alone interpret. It still remains to point out one important feature; if, as is rarely the case in practice, it is possible to allocate to the synchronous motor definite and continuous work, a comparatively small increment in its kilovolt-ampere rating will enable it to do much useful work, and thus considerably reduce the standing and running charges otherwise attributable to its installation.

For instance, if we join GE in Fig. 79 we find this length scales 1,373 units, and this represents the kilovolt-ampere rating of a synchronous motor capable of furnishing a leading wattless component of 1,114 kilovolt-amperes, and useful work to the extent represented by the length HE, namely, 700 kw. We thus see that by increasing the rating of the synchronous motor by 253 kilovolt-amperes, we enable it to do useful work to the extent of 700 kw.

As regards the effect of low-power factor upon the generating station itself, it will generally be found that it is cheaper to instal a larger generator than a synchronous motor. In some cases where the existing alternators have been unable to work up to the full load of the engines to which they were coupled, larger generators, but coupled to the same sized engines as before, were subsequently installed. By over-exciting the new generators the wattless current required was supplied by them,

and the whole of the steam plant thus enabled to work up to its full rated capacity.

Where the conditions are such that rotary converters can be employed, this plant possesses advantages over the synchronous motor as regards efficiency and reliability in working, and for a small increase in its kilovolt-ampere rating it will supply a very considerable leading wattless component for the correction of low-power factor.

Of recent years considerable attention has been given to the improvement of the induction motor, which may be said to be the chief cause of low-power factor upon existing supply systems. This has resulted in the development of commutating forms of induction motors in which the power factor may be made practically unity. With such motors a special exciter is combined, consisting of a commutating alternate current generator whose magnets are excited by the low frequency rotor currents.

Boosting.

It was pointed out in Chapter II. that the question as to whether it will pay to employ boosting upon one or more feeders leaving the generating station will depend upon the working cost of the boosting appliances employed, with the particular load curve to be met, as compared with the interest and sinking fund charges upon the cost of the extra copper which, if put into the feeders, would render boosting unnecessary.

The cost of attendance upon boosting appliances installed at the generating station ends of feeders will, in general, be slight, whereas, if installed at the substation ends of the feeders, the cost of attendance may become a considerable item.

As an example of boosting, we may take the case of a 2,000-volt single-phase feeder transmitting 300 kw. to a substation six miles distant, as follows :—

Distance of transmission	6 miles.
Load transmitted—Single-phase P.F. .1	300 kw.
Maximum bus bar pressure at generating station	2,200 volts.
Working pressure at substation	2,000 volts.
Full load current	150 amperes.
Load factor	13 per cent.

As alternative schemes we could adopt in this case :—

Scheme A (with Booster)—

One 0.15 sq. in. concentric cable with CR drop -	510 volts.
Less pressure supplied by booster - - - -	310 „
	200 „

Scheme B (without Booster)—

Two 0.2 sq. in. concentric cables with CR drop -	200 volts.

It is to be noted that under Scheme A the energy to be supplied by the booster is

$$310 \text{ volts} \times 150 \text{ amperes} = 46.7 \text{ kw.}$$

We may assume the following relative capital expenditure :—

Scheme A—

Six miles 0.15 sq. in. 2,200-volt V.B. C.C. cable laid in cast-iron trough, reinstatement 1st setts -	£9,054
50 kw. regulating booster, erected complete - -	150
	£9,204

Scheme B—

Twelve miles 0.2 sq. in. 2200-volt V.B. C.C. cable laid in cast-iron trough, reinstatement 1st setts	£16,956

The annual charges may now be set out as follows :—

TABLE XXI.

	With Booster.			Without Booster.		
Standing Charges.	£	s.	d.	£	s.	d.
Interest and sinking fund per annum on capital cost of cables laid complete, at 6 per cent. - - - - - - -	543	5	0	1,017	7	3
Interest and sinking fund per annum upon capital cost of boosting transformer and switchgear, at 6 per cent. -	9	0	0	...		
Interest and sinking fund at 6 per cent. upon proportion of generating plant representing losses in cables and boosting transformer at £35 per kw. installed - - - - - - -	162	12	0	63	0	0
Running Charges.						
Value of C^2R loss per annum in cables and boosting transformer at works, running cost of 0.375d. per unit - -	103	12	0	39	16	0
Value of open circuit losses, dielectric and copper losses in cables, iron and copper losses in boosting transformer, at works running cost of 0.375d. per unit - - - - - - -	7	8	5	5	4	0
	825	17	5	1,125	7	3

Saving per annum by adopting boosting, £299. 9s. 10d.

APPENDIX A

BOARD OF TRADE REGULATIONS FOR OVERHEAD WIRES

1. *Maximum Intervals between Supports.*—The interval between any two wooden poles used singly as supports for an overhead line shall not exceed 200 ft.; provided that where the line makes an angle at any such pole the interval between that and the next pole shall not exceed 150 ft. In the case of supports other than single wooden poles the intervals between the supports shall be such as may be prescribed by the Board of Trade.

2. *Factors of Safety.*—Every support for an overhead line shall be of a durable material, and shall be properly stayed against forces due to wind pressure, change of direction of the line, or unequal lengths of span. The factor of safety shall be for overhead lines at least 5, and for wooden poles at least 10, and for iron or steel structures at least 6, taking the maximum possible wind pressure at 50 lbs. per square foot. No addition need be made for a possible accumulation of snow.

3. *Attachment of Overhead Lines.*—All overhead lines shall be attached to insulators, and shall be so guarded that they cannot fall away from the support.

4. *Height from Ground, &c.*—An overhead line, placed after the date of these Regulations, shall not in any part thereof be at a less height from the ground than 22 ft., except with the consent of the Board of Trade, and shall not be accessible to any person without the use of a ladder or other special appliance; and, in the case of a high-pressure overhead line so placed, no part thereof which crosses a street shall be at a less height from the ground than 25 ft., except with the consent of the Board of Trade.

5. *Three-wire System.*—Where a supply is given by overhead lines on the three-wire system, the positive and negative conductors shall be placed side by side above the intermediate conductor. The intermediate conductor shall consist of two wires placed side by side at a distance apart greater than that between the positive and negative conductors, and connected in each span by two cross wires placed in such a manner that in the event of either the positive or negative conductor breaking, it shall fall on one at least of the cross wires.

6. *Supply from Two-wire System.*—Where a supply is given by overhead lines from a two-wire system, with the negative conductor connected with earth, the positive conductor shall be placed above the negative conductor in such a manner that in the event of breakage it must fall on the negative conductor.

7. *Service Lines from Overhead Lines.*—Service lines from overhead lines shall be led as directly as possible to insulators firmly attached to some portion of the consumer's premises which is not accessible to any person without the use of a ladder or other special appliance. Every portion of any service line which is outside a building but is within 7 ft. from the building, shall be efficiently protected by insulating material.

8. *Angle of Crossing Thoroughfares.*—Where an overhead line crosses a street, the angle between the line and the direction of the street at the place of crossing shall not be less than 60°, and the spans shall be as short as possible.

9. *Lines Crossing Metallic Substance.*—Where an overhead line crosses, or is in proximity to, any metallic substance, precautions shall be taken by the Undertakers against the possibility of the line coming into contact with the metallic substance, or of the metallic substance coming into contact with the line by breakage or otherwise.

10. *Discharge of Pressure in Case of Fire.*—In the case of any high-pressure overhead line exceeding one half mile in total length, means shall be provided whereby in case of fire or other emergency the pressure may be discharged from any portion of the line erected over or alongside of any building or buildings.

11. *Maintenance.*—Every overhead line, including its sup-

ports and all the structural parts and electrical appliances and devices belonging to or connected with the line, shall be duly and efficiently supervised and maintained as regards both electrical and mechanical conditions.

12. *Disused Overhead Lines to be Removed.*—The Undertakers shall remove any overhead line upon ceasing to use it for the supply of energy, unless upon so ceasing they satisfy the Board of Trade that they intend to bring it into use again within a reasonable time.

13. *Overhead Lines.*—Overhead lines shall not after the date of these Regulations be erected or maintained except in accordance with plans approved by the Board of Trade, and subject to such Regulations as the Board may prescribe; provided that this Regulation shall not apply to any electric lines which have been erected at the date of these Regulations so long as those lines are maintained in accordance with any Regulations of the Board of Trade which are in force and applicable thereto at that date, and with any requirements of the Board made thereunder.

These Regulations are made subject to the power of the Board of Trade to make such further or other Regulations as they may think expedient.

Consents for Overhead Wires.—Applications for consent for overhead wires are considered in each case on its merits The following information should be given :—

1. Where the Undertakers are a Company, evidence of consent of Local Authorities where the wires cross any road. The Local Authorities are :—(*a*) In England and Ireland: Borough Councils, Urban District Councils, and Rural District Councils; (*b*) In Scotland: Police Commissioners, Gas Commissioners, Town Councils, and County Councils.

2. A statement showing commercial or other considerations why underground cables should not be used.

3. A brief description of the proposed system.

4. Is the supply to form (1) an extension of an existing system or underground cables, or (2) of an existing traction system, or (3) an independent system?

5. A plan on a scale of about 6 in. to the mile, showing the proposed route of the overhead wire. (In the case of Ordnance maps, the sheets to be sent separately.)

6. In the case of high and extra high pressure, plans of construction of poles, &c., on a scale of about 1 in. to the foot.

Note.—Proposals for high or extra high-pressure wires carried alongside roads cannot be entertained, the transmission lines must go across open country.

APPENDIX B

SELF-INDUCTION OF WIRES

FROM purely theoretical considerations it may be shown that the self-induction in henrys per kilometre of two parallel wires L is given by the expression :—

$$L = \left(0.1 + 0.92 \log_{10} \frac{d}{r}\right) 10^{-3} \quad - \quad - \quad - \quad (1)$$

Where d = distance between the centres of the wires.
r = radius of the wires, r and d being expressed in similar units.

For a single core of a three-phase line this reduces to

$$L = \left(.05 + 0.46 \log_{10} \frac{d}{r}\right) 10^{-3} \quad - \quad - \quad - \quad (2)$$

The above expressions, however, make certain assumptions as to the current distribution in the conductors, which we know will depend upon the frequency of the current and the diameter of the conductors themselves.

If we write equation (2) in the form

$$L = \left(k + 0.46 \log_{10} \frac{d}{r}\right) 10^{-3}$$

we may assign to k certain values found to agree with actual measurements of self-induction under various working conditions.

With stranded cables of circular section insulated for 5,000 volts working pressure, it would appear that average values of k at normal frequencies are as follows :—

SECTION OF CONDUCTOR IN SQUARE INCH.	VALUE OF k.
0.025 sq. in.	.07
0.05 ,,	.05
0.1 ,,	.035
0.15 ,,	.025
0.2 ,,	.02

The self-induction of large cables formed of cores hammered to clover leaf section was found to be about 8 per cent. less than the corresponding values with circular cores.

APPENDIX C

ELECTROSTATIC CAPACITY OF WIRES

THE Y capacity K_1 per mile in microfarads of each conductor of a three-phase overhead line is given approximately by the following expression :—

$$K_1 = \frac{.0388}{\log_{10} \frac{d}{r}}$$

Where d = distance between centres of conductors.
r = radius of each conductor, r and d being expressed in similar units.

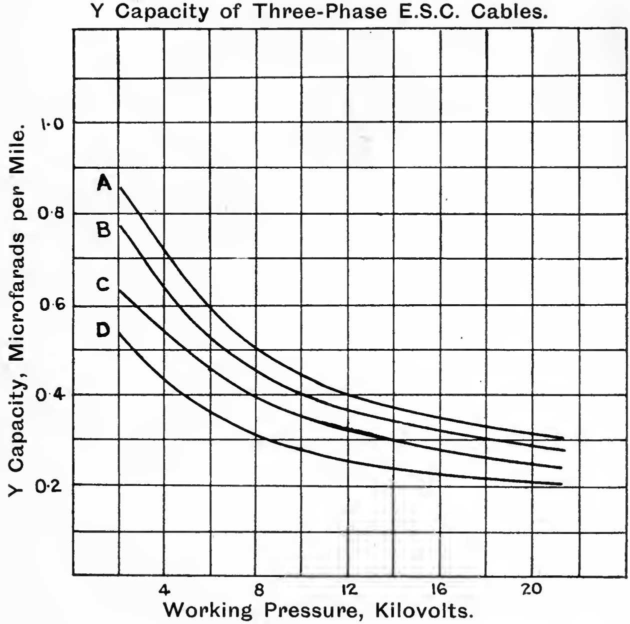
In the case of overhead lines, however, the distance the wires are from the ground will affect the electrostatic capacity of the line to some extent. If this be taken into account, we have for the Y capacity K_1 per mile of conductor in microfarads :—

$$K_1 = \frac{.0388}{\log_{10} \frac{d}{r} - \log_{10} \sqrt{1 + \left(\frac{d}{2h}\right)^2}}$$

Where d and r have the same meaning as before and h = mean height of wires above ground level, r, d, and h being expressed in similar units.

It was seen from Fig. 8, page 46, that the electrostatic capacity of a paper-insulated E.H.P. cable increased with rise in temperature. The capacity will also vary with the thickness and quality of the paper dielectric and hence with the working pressure for which the cable is constructed ; in addition it will vary with the section of the conductors themselves.

In Fig. 80 curves have been drawn illustrating the variation in Y capacity of paper-insulated three-phase cables constructed



A = 0.2 sq. in. B = 0.15 sq. in. C = 0.1 sq. in. D = .05 sq. in.

FIG. 80.

with Engineering Standards Committee's thickness of dielectric for various sections of conductor and working pressures.

APPENDIX D

LINE CALCULATIONS

THE solution of problems relating to transmission lines and cables carrying alternating currents due to impressed pressure waves of pure sine form can usually be effected very easily by graphical methods. Such methods possess the advantage also of rendering visible to the eye phase relationship and its changes due to reactance. The graphical method, however, in some cases results in incommensurable quantities, some lines when drawn to scale being inconveniently large and others as inconveniently small. An analytical method, generally known as the symbolic method, enables calculations regarding the most complicated combinations of alternate current circuits to be made with as much ease and accuracy as by simple arithmetic.

The method briefly explained involves two conventions as follows :—

(1) We know that the resultant of any number of pressures or currents, &c., may be found from a vector diagram as in the case of forces. The vectors may, however, be combined algebraically by adding the vertical and horizontal components of each vector, which will give the vertical and horizontal components of the resultant. To indicate that one set of components are at right angles to the initial line the prefix i is used. Thus if a voltage is given by the equation :—

$$V = RA + L\frac{\delta A}{\delta t}$$

it is represented by $V=(R+iLp)A$ in Fig. 81. If the coefficient of i is + then V is in advance of A, and the vector $iLpA$ is in advance of the vector RA.

Similarly, if

$$A_K = K\frac{\delta V}{\delta t} = iKpV,$$

the vector A_K is in advance of the vector V by 90°.

(2) If we choose lines along and perpendicular to OA as axes of reference, then V is represented by

$$V \cos \phi + iV \sin \phi.$$

If, however, our lines of reference are the axes *ox* and *oy*, and if the components of A upon these axes are respectively *a* and *b*, then the components of V are given by :—

$$V = [R + iLp]\,[a + ib].$$

Thus if we treat i^2 as $= -1$ we get

$$V = [aR - bLp] + i[bR + aLp].$$

As an example of the application of this method we may take the case of a three-phase inductive load requiring a constant

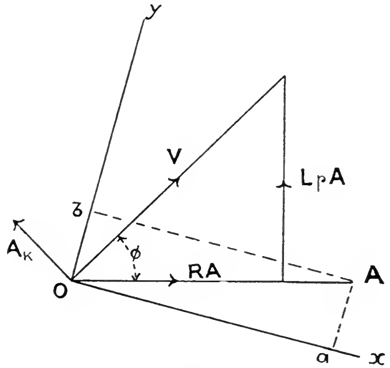


FIG. 81.

pressure of 20,000 volts between phases fed through a 50-mile length of .05 sq. in. three-core cable.

Assume—

Frequency of supply, 50 periods.
Power factor of load = 0.8.
Load current, 40 amperes per phase.

For a 50-mile length of .05 sq. in. three-core 20,000-volt cable we may take :—

Resistance per core -	=	42.95 ohms.
Inductance ($2lp$) per core	=	11.75 „
Y capacity per core -	=	10.55 microfarads.
Power factor of cable -	=	.028.

All three voltages and currents being equal, and the potentials of the system symmetrical with regard to earth, the problem reduces itself to a single-phase load of known power factor and constant pressure, fed through a single conductor of known resistance, self-induction, and capacity by a single-phase generator whose voltage is required, the circuit being completed through an imaginary earth return of resistance, &c., zero.

It will be sufficiently accurate for most practical purposes to consider the total capacity as bunched in the middle of the line

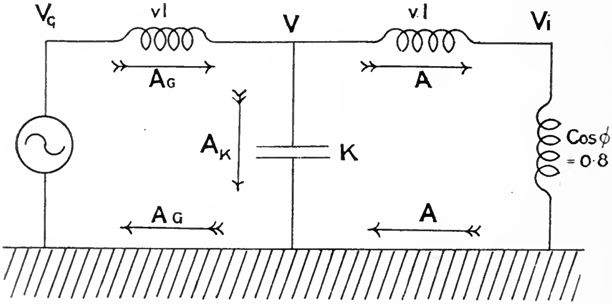


FIG. 82.

Our circuit may now be represented diagrammatically by Fig. 82, in which the values of our constants are as follows :—

$$V_L = \frac{20000}{\sqrt{3}} = 11{,}550 \text{ volts.}$$
$$r = 21.47 \text{ ohms.}$$
$$2\pi n l = 5.87 \text{ ohms.}$$
$$K = 10.55 \text{ microfarads.}$$
$$A = 40 \text{ amperes.}$$
$$\text{Cos } \phi = 0.8.$$

Starting with the required voltage at the load and working backwards towards the generator we have :—

$$\begin{aligned} V_L &= 11550 (\cos \phi + i \sin \phi) \\ &= 11550 (0.8 + i \,.\, 0.6) \\ &= 9240 + 6930 \,.\, i. \\ V - V_L &= [21.47 + 5.87 \,.\, i]\, 40 \\ &= 859 + 235 \,.\, i. \\ \therefore V &= 10.099 + 7165 \,.\, i. \end{aligned}$$

Now $$A_K = K\frac{\delta V}{\delta t} = 2\pi n KV \,.\, i,$$

and taking $$n = 50 \text{ periods},$$

$$2\pi n K = .00331.$$

Hence $$A_K = .00331 \,.\, i\,[10.099 + 7165 \,.\, i].$$

$$= -23.7 + 33.4 \,.\, i.$$

So far we have taken no account of the dielectric loss in the cable. With the assumed power factor of the cable .028 we must add to the above value of A_K a current:—

$$.028 \times .00331V = .936 + .665 \,.\, i.$$

Our value of A_K is now

$$-22.76 + 34.06 \,.\, i.$$

Now $$A_G = A_K + A$$

$$= 17.3 + 34.06 \,.\, i.$$

And as $$V_G - V = (r + i \,.\, 2\pi n l)\, A_G$$

$$= [21.47 + 5.87 \,.\, i]\,[17.3 + 34.06 \,.\, i]$$

$$= 172 + 833 \,.\, i.$$

Therefore $$V_G = 10.271 + 7998 \,.\, i.$$

We have now obtained the components of the vectors representing the various voltages and currents, and the length of each vector is at once arrived at by taking the square root of the sum of the squares of its components.

Thus $$V_G = \sqrt{(10,271)^2 + (7998)^2}$$

$$= 13,020 \text{ volts.}$$

Similarly $$A_G = 38.2 \text{ amperes.}$$

Similarly $$V = 12,390 \text{ volts.}$$

Again the tangent of the angle each vector makes with the initial line is given by the ratio of the vertical to the horizontal components.

Thus for V_G $\tan\phi = \dfrac{7998}{10,271} = .779$ and $\phi = 38^\circ$ nearly.

The power due to a voltage and a current vector is given in watts by adding the product of their respective horizontal components to the product of their vertical components.

Thus the power due to V_G and A_G is—

$$10,271 \times 17.3 + 7998 \times 34.06 = 450.2 \text{ kw. per phase,}$$

and the power factor $= \cos 25^\circ = .906$.

Proceeding after the manner indicated above, a number of results have been worked out and are plotted in the curves given by Fig. 11, which relate to a three-phase transmission under the working conditions assumed.

APPENDIX E

COMPARISON OF TRANSMISSION SYSTEMS

We have seen from Chapter III. that in transmitting a certain amount of power with three-phase cables over a given distance, the sectional area of the conductors or the amount of copper required will vary inversely as the square of the pressure employed for the same loss in the line. A similar result may be shown to be true generally with all transmission systems. As, however, different potentials will often occur between parts of the same transmission system, it becomes of importance in making any comparison to arrive at definite pressure relations with regard to each system as a basis for such comparison. Thus the amount of copper required to transmit a definite amount of power over a given distance, and with a fixed loss in the line, by any particular system, may be considered on the basis of equal maximum pressures between any parts of the system, or, on the other hand, that of equal maximum pressure between any parts of the system and earth. Further, the minimum pressures between the branches of each system may be taken as a basis for comparison.

In making comparisons between direct current and alternating current systems, it is also to be noted that whilst in the case of the direct current system the maximum pressure will not exceed the working pressure between parts of the system, yet in the case of alternating current systems the maximum pressure will be $\sqrt{2}$ times as great as the effective working pressure between the same parts of the system. The continuous current system would, therefore, on the basis of equal maximum pressure, only be expected to require half the amount of copper as the alternating current system.

With the transmission of power over long distances the most important basis for the comparison of the different systems is

the maximum potential, which will occur between any part of the system.

Starting with the single-phase system let

e = effective voltage between wires.
c = effective current per wire.
r = resistance per wire.

Then the power transmitted $= ec$,
and the line loss $= 2c^2r$.

With a two-phase system having four wires we have two distinct single-phase circuits, each wire carrying half the current and having twice the resistance of the single-phase conductors, the amount of copper required will be the same for equal loss and power transmitted.

In the case of a two-phase three-wire system, the pressure between each outer and common return must be reduced to $\frac{e}{\sqrt{2}}$ to maintain the same maximum pressure e between wires, the current in each outer will be $\frac{c}{\sqrt{2}}$ and in the common return c.

If we denote by r_1, the resistance of each outer, the total resistance of the conductors will be

$$2r_1 + \frac{r_1}{\sqrt{2}}$$

and the total loss

$$c^2r_1\frac{(2+\sqrt{2})}{2};$$

equating this to the loss in the single-phase system we find

$$\frac{r_1}{r} = \frac{4}{2+\sqrt{2}}$$

and the amount of copper required is 1.457 times that of the single-phase system.

With a three-phase system employing three wires with pressure e between them and line current c_1, we have :—

Power transmitted $= ec_1\sqrt{3}$.
Line loss $= 3c_1^2r_1$.

Now, since $ec_1\sqrt{3}$ must be equal to the power of the single-phase system ec; $c_1 = \frac{c}{\sqrt{3}}$, and the total loss will be c^2r_1 as com-

pared with a loss of $2c^2r$ in the single-phase system. Hence, $r_1 = 2r$, and the amount of copper required in each three-phase line is only half of that in each single-phase line. The total respective weights of copper will, therefore, be as $\frac{3}{2}$ to 2 or $\frac{3}{4}$ to 1. That is, the three-phase system will only require three-quarters of the copper of the single-phase system.

Collecting the above results and taking the weight of copper required by the single-phase circuit as 100, we have the following relative values :—

TABLE XXII.

System.	No. of Wires.	Relative Weight of Copper.
Two-Phase - -	3	145.7
Two-Phase - -	4	100
One-Phase - -	2	100
Three-Phase -	3	75

Proceeding as before, however, we may compare the above systems as regards the same maximum potentials to earth, in which case we shall find some considerable differences, as will be seen from the following table :—

TABLE XXIII.

System.	No. of Wires.	Relative Weight of Copper.
One-Phase, one pole earthed - -	2	100
Two-Phase, one pole earthed - -	4	100
Two-Phase, common return earthed	3	72.8
Three-Phase, neutral point earthed	3	25

INDEX

Printed at THE DARIEN PRESS, *Edinburgh.*

ATKINSON, A. A. **Electrical and Magnetic Calculations.** For the use of Electrical Engineers and others interested in the Theory andApplication of Electricity and Magnetism. *Third Edition, revised.* Illustrated. 8vo., cloth, 310 pp. ..Net, $1.50

ATKINSON, PHILIP. **The Elements of Dynamic Electricity and Magnetism.** *Fourth Edition.* Illustrated. 12mo., cloth, 405 pp....................$2.00

Elements of Electric Lighting, including Electric Generation, Measurement, Storage, and Distribution. *Tenth Edition,* fully revised and new matter added. Illustrated. 12mo., cloth, 280 pp....................$1.50

Power Transmitted by Electricity and Applied by the Electric Motor, including Electric Railway Construction. Illustrated. *Fourth Edition,* fully revised and new matter added. 12mo., cloth, 241 pp....................$2.00

AYRTON, HERTHA. **The Electric Arc.** Illus. 8vo., cloth, 479 pp....Net $5.00

—— **W. E.** **Practical Electricity.** A Laboratory and Lecture Course. Illustrated. 12mo., cloth, 643 pp....................$2.00

BAKER, J. T. **The Telegraphic Transmission of Photographs.** 63 illustrations. 12mo., cloth, 155 pp....................Net, $1.25

BEDELL, FREDERICK. **Direct and Alternating Current Testing.** Assisted by C. A. Pierce. Illustrated. 8vo., cloth, 250 pp....................Net, $2.00

BEDELL, F. & CREHORE, ALBERT C. **Alternating Currents.** An analytical and graphical treatment for students and engineers. *Fifth Edition.* 112 illustrations. 8vo., cloth, 325 pp....................Net, $2.50

BLAINE, ROBERT G. **The Calculus and Its Applications.** A practical treatise for beginners especially engineering students. 79 illustrations. 12mo., cloth, 330 pp....................Net $1.50

BIGGS, C. H. W. **First Principles of Electricity and Magnetism.** Illustrated. 12mo., cloth. 495 pp....................$2.00

BONNEY, G. E. **The Electro-Plater's Hand Book.** A Manual for Amateurs and Young Students on Electro-Metallurgy. *Fourth Edition, enlarged.* 61 Illustrations. 12mo., cloth, 208 pp....................$1.20

BOTTONE, S. R. **Magnetos For Automobilists, How Made and How Used.** A handbook of practical instruction on the manufacture and adaptation of the magneto to the needs of the motorist. *Second Edition, enlarged.* 52 Illustrations. 12mo., cloth, 188p pp....................Net, $1.00

Electric Motors, How Made and How Used. Illustrated. 12mo., cloth, 168 pp....................75 cents

BOWKER, WM. R. **Dynamo, Motor, and Switchboard Circuits for Electrical Engineers:** A practical book dealing with the subject of direct, alternating, and polyphase circuits. *Second Edition, greatly enlarged.* 130 illustrations. 8vo., cloth, 180 pp....................Net, $2.50

CARTER, E. T. Motive Power and Gearing for Electrical Machinery; a treatise on the theory and practice of the mechanical equipment of power stations for electric supply and for electric traction. *Second Edition, revised.* Illustrated. 8vo., cloth, 700 pp. . . . . .Net, $5.00

CHILD, CHAS. T. The How and Why of Electricity: a book of information for non-technical readers, treating of the properties of Electricity, and how it is generated, handled, controlled, measured, and set to work. Also explaining the operation of Electrical Apparatus. Illustrated. 8vo., cloth, 140 pp. $1 00

CLARK, D. K. Tramways, Their Construction and Working. *Second Edition.* Illustrated. 8vo., cloth, 758 pp. $9.00

COOPER, W. R. Primary Batteries: their Theory, Construction, and Use. 131 Illustrations. 8vo., cloth, 324 pp. . . . . .Net, $4.00

The Electrician Primers. Being a series of helpful primers on electrical subjects, for the use of students, pupils, artisans, and general readers. *Second Edition.* Illustrated. Three volumes in one. 8vo., cloth, Net, $5.00

Vol. I.—Theory . . . . .Net, $2.00

Vol. II.—Electric Traction, Lighting and Power. . . . . .Net, $3.00

Vol. III.—Telegraphy, Telephony, etc. . . . . .Net, $2.00

CROCKER, F. B. Electric Lighting. A Practical Exposition of the Art for the use of Electricians, Students, and others interested in the Installation or Operation of Electric-Lighting Plants.

Vol. I.—The Generating Plant. *Sixth Edition, entirely revised.* Illustrated. 8vo., cloth, 482 pp. $3.00

Vol. II.—Distributing System and Lamps. *Sixth Edition.* Illustrated. 8vo., cloth, 505 pp. $3.00

—— and ARENDT, M. Electric Motors: Their Action, Control, and Application. 160 Illustrations. 8vo., cloth, 296 pp. . . . . .Net, 2.50

—— and WHEELER, S. S. The Management of Electrical Machinery. Being a *thoroughly revised and rewritten edition* of the authors' "Practical Management of Dynamos and Motors." *Eighth Edition.* Illustrated. 16mo., cloth, 232 pp. . . . . .Net, $1.00

CUSHING, H. C., Jr. Standard Wiring for Electric Light and Power. Illustrated. 16mo., leather, 156 pp. $1.00

DAVIES, F. H. Electric Power and Traction. Illustrated. 8vo., cloth, 293 pp (Van Nostrand's Westminster Series.) . . . . .Net, $2.00

DAWSON, PHILIP. Electric Traction on Railways. 610 Illustrations. 8vo., half leather, 891 pp. . . . . .Net, $9.00

DEL MAR, W. A. Electric Power Conductors. 69 illustrations. 8vo., cloth, 330 pp. Net, $2.00

DINGER, Lieut. H. C. Handbook for the Care and Operation of Naval Machinery. *Second Edition.* Illustrated. 16mo., cloth, 302 pp. . . . . .Net, $2.00

DYNAMIC ELECTRICITY; Its Modern Use and Measurement, chiefly in its application to Electric Lighting and Telegraphy, including: 1. Some Points in Electric Lighting, by Dr. John Hopkinson. 2. On the Treatment of Electricity for Commercial Purposes, by J. N. Shoolbred. 3. Electric-Light Arithmetic, by R. E. Day, M.E. *Fourth Edition.* Illustrated. 16mo., boards, 166 pp. (No. 71 Van Nostrand's Science Series.)......50 cents

EDGCUMBE, K. Industrial Electrical Measuring Instruments. Illustrated. 8vo., cloth, 227 pp...Net, $2.50

ERSKINE-MURRAY, J. A Handbook of Wireless Telegraphy: Its Theory and Practice. For the use of electrical engineers, students, and operators. *Second Edition, revised and enlarged.* 180 illustrations. 8vo., cloth, 388 pp...Net, $3.50

—— Wireless Telephones and How they Work. Illustrated. 16mo., cloth, 75 pp. $1.00

EWING, J. A. Magnetic Induction in Iron and other Metals. *Third Edition, revised.* Illustrated. 8vo., cloth, 393 pp.............................Net $4.00

FISHER, H. K. C., and DARBY, W. C. Students' Guide to Submarine Cable Testing. *Third Edition, enlarged.* Illus. 8vo., cloth, 326 pp......Net, $3.50

FLEMING, J. A. The Alternate-Current Transformer in Theory and Practice.

Vol. I.: The Induction of Electric Currents. *Fifth Issue.* Illustrated. 8vo., cloth, 641 pp...Net, $5.00

Vol. II.: The Utilization of Induced Currents. *Third Issue.* Illustrated. 8vo, cloth, 587 pp...Net, $5.00

Handbook for the Electrical Laboratory and Testing Room. Two Volumes. Illustrated. 8vo., cloth, 1160 pp. Each vol.................Net, $5.00

FOSTER, H. A. With the Collaboration of Eminent Specialists. Electrical Engineers' Pocket Book. A handbook of useful data for Electricians and Electrical Engineers. With innumerable Tables, Diagrams, and Figures. The most complete book of its kind ever published, treating of the latest and best Practice in Electrical Engineering. *Fifth Edition, completely revised and enlarged.* Fully Illustrated. Pocket Size. Leather. Thumb Indexed. 1636 pp...$5.00

FOWLE, F. F. The Protection of Railroads from Overhead Transmission Line Crossings. 35 illustrations. 12mo., cloth, 76 pp................Net, $1.50

GANT, L. W. Elements of Electric Traction for Motormen and Others. Illustrated with Diagrams. 8vo., cloth, 217 pp.........................Net, $2.50

GERHARDI, C. H. W. Electricity Meters; their Construction and Management. A practical manual for central station engineers, distribution engineers and students. Illustrated. 8vo., cloth, 337 pp................Net, $4.00

GORE, GEORGE. **The Art of Electrolytic Separation of Metals (Theoretical and Practical).** Illustrated. 8vo., cloth, 295 pp..................Net, $3.50

GRAY, J. **Electrical Influence Machines: Their Historical Development and Modern Forms.** With Instructions for making them. *Second Edition, revised and enlarged.* With 105 Figures and Diagrams. 12mo., cloth, 296 pp..$2.00

GROTH, L. A. **Welding and Cutting Metals by Aid of Gases or Electricity.** 124 illustrations. 8vo., cloth, 280 pp..............................Net, $3.00

HAMMER, W. J. **Radium, and Other Radio-Active Substances; Polonium, Actinium, and Thorium.** With a consideration of Phosphorescent and Fluorescent Substances, the properties and applications of Selenium, and the treatment of disease by the Ultra-Violet Light. With Engravings and Plates. 8vo., cloth, 72 pp....................................$1.00

HARRISON, N. **Electric Wiring Diagrams and Switchboards.** Illustrated. 12mo., cloth, 272 pp..$1.50

HASKINS, C. H. **The Galvanometer and its Uses.** A Manual for Electricians and Students. *Fifth Edition, revised.* Illus. 16mo., morocco, 75 pp..$1.50

HAY, ALFRED. **Principles of Alternate-Current Working.** *Second Edition.* Illustrated. 12mo., cloth, 390 pp.................................$2.00

Alternating Currents; their theory, generation, and transformation. *Second Edition.* 1178 Illustrations. 8vo., cloth, 319 pp..............Net, $2.50

An Introductory Course of Continuous-Current Engineering. Illustrated. 8vo., cloth, 327 pp..Net, $2.50

HEAVISIDE, O. **Electromagnetic Theory.** Two Volumes with Many Diagrams. 8vo., cloth, 1006 pp. Each vol..............................Net, $5.00

HEDGES, K. **Modern Lightning Conductors.** An illustrated Supplement to the Report of the Research Committee of 1905, with notes as to methods of protection and specifications. Illustrated. 8vo., cloth, 119 pp..Net, $3.00

HOBART, H. M. **Heavy Electrical Engineering.** Illustrated. 8vo., cloth, 338 pp...Net, $4.50

—— **Electricity.** A text-book designed in particular for engineering students. 115 illustrations. 43 tables. 8vo., cloth, 266 pp................Net, $2.00

HOBBS, W. R. P. **The Arithmetic of Electrical Measurements.** With numerous examples, fully worked. *Twelfth Edition.* 12mo., cloth 126 pp...50 cents

HOMANS, J. E. **A B C of the Telephone.** With 269 Illustrations. 12mo., cloth, 352 pp..$1.00

HOPKINS, N. M. **Experimental Electrochemistry, Theoretically and Practically Treated.** Profusely illustrated with 130 new drawings, diagrams, and photographs, accompanied by a Bibliography. Illustrated. 8vo., cloth, 298 pp...Net, $3.00

HOUSTON, EDWIN J. A Dictionary of Electrical Words, Terms, and Phrases. *Fourth Edition, rewritten and greatly enlarged.* 582 Illustrations. 4to., cloth...Net, $7.00

A Pocket Dictionary of Electrical Words, Terms, and Phrases. 12mo., cloth, 950 pp...Net, $2.50

HUTCHINSON, R. W., Jr. Long-Distance Electric Power Transmission: Being a Treatise on the Hydro-Electric Generation of Energy; Its Transformation, Transmission, and Distribution. *Second Edition.* Illustrated. 12mo., cloth, 350 pp...Net, $3.00

—— **and IHLSENG, M. C.** Electricity in Mining. Being a theoretical and practical treatise on the construction, operation, and maintenance of electrical mining machinery. Illustrated. 12mo., cloth..................*In Press*

INCANDESCENT ELECTRIC LIGHTING. A Practical Description of the Edison System, by H. Latimer. To which is added: The Design and Operation of Incandescent Stations, by C. J. Field; A Description of the Edison Electrolyte Meter, by A. E. Kennelly; and a Paper on the Maximum Efficiency of Incandescent Lamps, by T. W. Howell. *Fifth Edition.* Illustrated. 16mo., cloth, 140 pp. (No. 57 Van Nostrand's Science Series.)....50 cents

INDUCTION COILS: How Made and How Used. *Eleventh Edition.* Illustrated. 16mo., cloth, 123 pp. (No. 53 Van Nostrand's Science Series.)...50 cents

JEHL, FRANCIS. The Manufacture of Carbons for Electric Lighting and Other Purposes. Illustrated. 8vo , cloth, 232 pp....................Net, $4.00

JONES, HARRY C. The Electrical Nature of Matter and Radioactivity. *Second Edition, completely revised.* 12mo., cloth, 212 pp......................$2.00

KAPP, GISBERT. Electric Transmission of Energy and its Transformation, Subdivision, and Distribution. A Practical Handbook. *Fourth Edition, thoroughly revised.* Illustrated. 12mo., cloth, 445 pp..............$3.50

KAPP, GISBERT. Alternate-Current Machinery. Illustrated. 16mo., cloth, 190 pp. (No. 96 Van Nostrand's Science Series.)..............50 cents

Dynamos, Alternators, and Transformers. Illustrated. 8vo., cloth, 507 pp...$4.00

KELSEY, W. R. Continuous-Current Dynamos and Motors, and their Control; being a series of articles reprinted from the "Practical Engineer," and completed by W R. Kelsey, B.Sc. With Tables, Figures, and Diagrams. 8vo., cloth, 439 pp...$2.50

KEMPE, H. R. A Handbook of Electrical Testing. *Seventh Edition, revised and enlarged.* 285 Illustrations. 8vo., cloth, 706 pp....Net, $6.00

KENNEDY, R. Modern Engines and Power Generators. Illustrated. 4to.,

Electrical Installations of Electric Light, Power, and Traction Machinery. Illustrated. 8vo., cloth, 5 vols. The Set, $15.00............. Each, $3.50

KENNELLY, A. E. Theoretical Elements of Electro-Dynamic Machinery. Vol I. Illustrated. 8vo., cloth, 90 pp. $1.50

KERSHAW, J. B. C. The Electric Furnace in Iron and Steel Production. Illustrated. 8vo., cloth, 74 pp. . . . . Net, $1.50

Electrometallurgy. Illustrated. 8vo., cloth, 303 pp. (Van Nostrand's Westminster Series.) . . . . Net, $2.00

KINZBRUNNER, C. Continuous-Current Armatures; their Winding and Construction. 79 Illustrations. 8vo., cloth, 80 pp. . . . . Net, $1.50

Alternate-Current Windings; their Theory and Construction. 89 Illustrations. 8vo., cloth, 80 pp. . . . . Net, $1.50

KOESTER, F. Hydroelectric Developments and Engineering. A practical and theoretical treatise on the development, design, construction, equipment and operation of hydroelectric transmission plants. 500 illustrations. 4to., cloth, 475 pp. . . . . Net, $5.00

—— **Steam-Electric Power Plants.** A practical treatise on the design of central light and power stations and their economical construction and operation. Fully Illustrated. 4to., cloth, 455 pp . . . . Net, $5.00

LARNER, E. T. The Principles of Alternating Currents for Students of Electrical Engineering. Illustrated with Diagrams. 12mo., cloth, 144 pp. Net, $1.50

LEMSTROM, S. Electricity in Agriculture and Horticulture. Illustrated. 8vo., cloth. . . . . Net, $1.50

LIVERMORE, V. P., and WILLIAMS, J. How to Become a Competent Motorman: Being a practical treatise on the proper method of operating a street-railway motor-car; also giving details how to overcome certain defects. *Second Edition.* Illustrated. 16mo., cloth, 247 pp. . . . . Net, $1.00

LOCKWOOD, T. D. Electricity, Magnetism, and Electro-Telegraphy. A Practical Guide and Handbook of General Information for Electrical Students, Operators, and Inspectors. *Fourth Edition.* Illustrated. 8vo., cloth, 374 pp. $2.50

LODGE, OLIVER J. Signalling Across Space Without Wires: Being a description of the work of Hertz and his successors. *Fourth Edition.* Illustrated. 8vo., cloth, 156 pp. . . . . Net, $2.00

LORING, A. E. A Handbook of the Electro-Magnetic Telegraph. *Fourth Edition, revised.* Illustrated. 16mo., cloth, 116 pp. (No. 39 Van Nostrand's Science Series.) . . . . 50 cents

LUPTON, A. PARR, G. D. A., and PERKIN, H. Electricity Applied to Mining. *Second Edition.* With Tables, Diagrams, and Folding Plates. 8vo., cloth, 320 pp. . . . . Net, $4.50

MAILLOUX, C. O. Electric Traction Machinery. Illustrated. 8vo., cloth. *In Press*

MANSFIELD, A. N. Electromagnets: Their Design and Construction. *Second Edition.* Illustrated. 16mo., cloth, 155 pp. (Van Nostrand's Science Series No. 64.)..50 cents

MASSIE, W. W., and UNDERHILL, C. R. **Wireless Telegraphy and Telephony Popularly Explained.** Illustrated. 12mo., cloth, 82 pp.........Net, $1.00

MAURICE, W. **Electrical Blasting Apparatus and Explosives,** with special reference to colliery practice. Illustrated. 8vo., cloth, 167 pp......Net, $3.50

MONCKTON, C. C. F. **Radio Telegraphy.** 173 Illustrations. 8vo., cloth, 272 pp. (Van Nostrand's Westminster Series.)........................Net, $2.00

MORGAN, ALFRED P. **Wireless Telegraph Construction for Amateurs.** 153 illustrations. 12mo., cloth, 220 pp..............................Net, $1.50

NIPHER, FRANCIS E. **Theory of Magnetic Measurements.** With an Appendix on the Method of Least Squares. Illustrated. 12mo., cloth, 94 pp...$1.00

NOLL, AUGUSTUS. **How to Wire Buildings.** A Manual of the Art of Interior Wiring. *Fourth Edition.* Illustrated. 12mo., cloth, 165 pp........$1.50

OHM, G. S. **The Galvanic Circuit Investigated Mathematically.** Berlin, 1827. Translated by William Francis. With Preface and Notes by the Editor, Thos. D. Lockwood. *Second Edition.* Illustrated. 16mo., cloth, 269 pp. (No. 102 Van Nostrand's Science Series.)......................50 cents

OLSSON, ANDREW. **Motor Control as Used in Connection with Turret Turning and Gun Elevating** (The Ward Leonard System.) 13 illustrations. 12mo., paper, 27 pp. (U. S. Navy Electrical Series No. 1.)...............Net, .50

OUDIN, MAURICE A. **Standard Polyphase Apparatus and Systems.** Illustrated with many Photo-reproductions, Diagrams, and Tables. *Fifth Edition, revised.* 8vo., cloth, 369 pp.....................................Net, $3.00

PALAZ, A. **Treatise on Industrial Photometry.** Specially applied to Electric Lighting. Translated from the French by G. W. Patterson, Jr., Assistant Professor of Physics in the University of Michigan, and M. R. Patterson, B.A. *Second Edition.* Fully Illustrated. 8vo., cloth, 324 pp......$4.00

PARR, G. D. A. **Electrical Engineering Measuring Instruments for Commercial and Laboratory Purposes.** With 370 Diagrams and Engravings. 8vo., cloth, 328 pp..Net, $3.50

PARSHALL, H. F., and HOBART, H. M. **Armature Windings of Electric Machines.** *Third Edition.* With 140 full-page Plates, 65 Tables, and 165 pages of descriptive letter-press. 4to., cloth, 300 pp.......................$7.50

Electric Railway Engineering. With 437 Figures and Diagrams and many Tables. 4to., cloth, 475 pp..............................Net, $10.00

Electric Machine Design. Being a revised and enlarged edition of "Electric Generators." 648 Illustrations. 4to, half morocco, 601 pp....Net, $12.50

PERRINE, F. A. C. Conductors for Electrical Distribution: Their Manufacture and Materials, the Calculation of Circuits, Pole-Line Construction, Underground Working, and other Uses. *Second Edition.* Illustrated. 8vo., cloth, 287 pp. . . . . .Net, $3.50

POPE, F. L. Modern Practice of the Electric Telegraph. A Handbook for Electricians and Operators. *Seventeenth Edition.* Illustrated. 8vo., cloth, 234 pp. $1.50

RAPHAEL, F. C. Localization of Faults in Electric-Light Mains. *Second Edition, revised.* Illustrated. 8vo., cloth, 205 pp. .Net, $3.00

RAYMOND, E. B. Alternating-Current Engineering, Practically Treated. *Third Edition, revised.* With many Figures and Diagrams. 8vo., cloth, 244 pp., Net, $2.50

RICHARDSON, S. S. Magnetism and Electricity and the Principles of Electrical Measurement. 254 illustrations. 12mo., cloth, 604 pp. . . . . .Net, $2.00

ROBERTS, J. Laboratory Work in Electrical Engineering—Preliminary Grade. A series of laboratory experiments for first- and second-year students in electrical engineering. Illustrated with many Diagrams. 8vo., cloth, 218 pp. . . . . .Net, $2.00

RUHMER, ERNST. Wireless Telephony in Theory and Practice. Translated from the German by James Erskine-Murray. Illustrated. 8vo., cloth, 224 pp. . . . . .Net, $3.50

RUSSELL, A. The Theory of Electric Cables and Networks. 71 Illustrations. 8vo., cloth, 275 pp. . . . . .Net, $3.00

SALOMONS, DAVID. Electric-Light Installations. A Practical Handbook. Illustrated. 12mo., cloth.

Vol I.: **Management of Accumulators.** *Ninth Edition.* 178 pp. $2.50
Vol. II.: **Apparatus.** *Seventh Edition.* 318 pp. $2.25
Vol. III.: **Application.** *Seventh Edition.* 234 pp. $1.50

SEVER, G. F. Electrical Engineering Experiments and Tests on Direct-Current Machinery. *Second Edition, enlarged.* With Diagrams and Figures. 8vo., pamphlet, 75 pp. . . . . .Net, $1.00

—— and TOWNSEND, F. Laboratory and Factory Tests in Electrical Engineering. *Second Edition.* Illustrated. 8vo., cloth, 269 pp. . . . . .Net, $2.50

SEWALL, C. H. Wireless Telegraphy. With Diagrams and Figures. *Second Edition, corrected.* Illustrated. 8vo., cloth, 229 pp. . . . . .Net, $2.00

Lessons in Telegraphy. Illustrated. 12mo., cloth, 104 pp. . . . . . . .Net, $1.00

—— T. Elements of Electrical Engineering. *Third Edition, revised.* Illustrated. 8vo., cloth, 444 pp. $3.00

The Construction of Dynamos (Alternating and Direct Current). A Textbook for students, engineering contractors, and electricians-in-charge. Illustrated. 8vo., cloth, 316 pp. $3.00

SHELDON, S., and HAUSMANN, E. Dynamo-Electric Machinery: Its Construction, Design, and Operation.

Vol. I.: Direct-Current Machines. *Eighth Edition, completely rewritten.* Illustrated. 8vo., cloth, 281 pp.............................Net, $2.50

—— **and MASON, H. Alternating-Current Machines:** Being the second volume of "Dynamo-Electric Machinery; its Construction, Design, and Operation." With many Diagrams and Figures. (Binding uniform with Volume I.) *Seventh Edition, rewritten.* 8vo., cloth, 353 pp........Net, $2.50

SLOANE, T. O'CONOR. Standard Electrical Dictionary. 300 Illustrations. 12mo., cloth, 682 pp...$3.00

Elementary Electrical Calculations. A Manual of Simple Engineering Mathematics, covering the whole field of Direct Current Calculations, the basis of Alternating Current Mathematics, Networks, and typical cases of Circuits, with Appendices on special subjects. 8vo., cloth. Illustrated. 304 pp..Net, $2.00

SNELL, ALBION T. Electric Motive Power. The Transmission and Distribution of Electric Power by Continuous and Alternating Currents. With a Section on the Applications of Electricity to Mining Work. *Second Edition.* Illustrated. 8vo., cloth, 411 pp... ....Net, $4.00

SODDY, F. Radio-Activity; an Elementary Treatise from the Standpoint of the Disintegration Theory. Fully Illustrated. 8vo., cloth, 214 pp..Net, $3.00

SOLOMON, MAURICE. Electric Lamps. Illustrated. 8vo., cloth. (Van Nostrand's Westminster Series.)...............................Net, $2.00

STEWART, A. Modern Polyphase Machinery. Illustrated. 12mo., cloth, 296 pp..Net, $2.00

SWINBURNE, JAS., and WORDINGHAM, C. H. The Measurement of Electric Currents. Electrical Measuring Instruments. Meters for Electrical Energy. Edited, with Preface, by T. Commerford Martin. Folding Plate and Numerous Illustrations. 16mo., cloth, 241 pp. (No. 109 Van Nostrand's Science Series.)...50 cents

SWOOPE, C. WALTON. Lessons in Practical Electricity: Principles, Experiments, and Arithmetical Problems. An Elementary Text-book. *Eleventh Edition, enlarged with a chapter on alternating currents.* 404 illustrations. 12mo., cloth, 507 pp.......................................Net, $2.00

THOM, C., and JONES, W. H. Telegraphic Connections, embracing recent methods in Quadruplex Telegraphy. 20 Colored Plates. 8vo., cloth, 59 pp..$1.50

THOMPSON, S. P., Prof. Dynamo-Electric Machinery. With an Introduction and Notes by Frank L. Pope and H. R. Butler. Fully Illustrated. 16mo., cloth, 214 pp. (No. 66 Van Nostrand's Science Series.)...........50 cents

Recent Progress in Dynamo-Electric Machines. Being a Supplement to "Dynamo-Electric Machinery." Illustrated. 16mo., cloth, 113 pp. (No. 75 Van Nostrand's Science Series.)............................50 cents

TOWNSEND, FITZHUGH. Alternating Current Engineering. Illustrated. 8vo., paper, 32 pp..Net, 75 cents

UNDERHILL, C. R. Solenoids, Electromagnets and Electromagnetic Windings. 218 Illustrations. 12mo., cloth, 345 pp..........................Net, $2.00

URQUHART, J. W. Dynamo Construction. A Practical Handbook for the use of Engineer Constructors and Electricians in Charge. Illustrated. 12mo., cloth..$3.00

Electric Ship-Lighting. A Handbook on the Practical Fitting and Running of Ship's Electrical Plant, for the use of Ship Owners and Builders, Marine Electricians, and Sea-going Engineers in Charge. 88 Illustrations. 12mo., cloth, 308 pp..$3.00

Electric-Light Fitting. A Handbook for Working Electrical Engineers, embodying Practical Notes on Installation Management. *Second Edition.* With numerous Illustrations. 12mo., cloth........................$2.00

Electroplating. *Fifth Edition.* Illustrated. 12mo., cloth, 230 pp......$2.00

Electrotyping. Illustrated. 12mo., cloth, 228 pp....................$2.00

WADE, E. J. Secondary Batteries: Their Theory, Construction, and Use. *Second Edition, corrected.* 265 illustrations. 8vo, cloth, 501 pp........Net, $4.00

WADSWORTH, C. Electric Ignition. 15 Illustrations. 16mo. paper. *In Press*

WALKER, FREDERICK. Practical Dynamo-Building for Amateurs. How to Wind for any Output. *Third Edition.* Illustrated. 16mo., cloth, 104 pp. (No. 98 Van Nostrand's Science Series.)......................50 cents

—— SYDNEY F. Electricity in Homes and Workshops. A Practical Treatise on Auxiliary Electrical Apparatus. *Fourth Edition.* Illustrated. 12mo., cloth, 358 pp...$2.00

Electricity in Mining. Illustrated. 8vo., cloth, 385 pp...............$3.50

WALLING, B. T., Lieut.-Com. U.S.N., and MARTIN, JULIUS. Electrical Installations of the United States Navy. With many Diagrams and Engravings. 8vo., cloth, 648 pp...$6.00

WATT, ALEXANDER. Electroplating and Refining of Metals. *New Edition,* rewritten by Arnold Philip. Illustrated. 8vo., cloth, 677 pp..Net, $4.50

Electro-Metallurgy. *Fifteenth Edition.* Illustrated. 12mo., cloth, 225 pp., $1.00

WEBB, H. L. A Practical Guide to the Testing of Insulated Wires and Cables. *Fifth Edition.* Illustrated. 12mo., cloth, 118 pp.................$1.00

WEEKS, R. W. The Design of Alternate-Current Transformer. *New Edition in Press*

WEYMOUTH, F. MARTEN. **Drum Armatures and Commutators.** (Theory and Practice.) A complete treatise on the theory and construction of drum-winding, and of commutators for closed-coil armatures, together with a full résumé of some of the principal points involved in their design, and an exposition of armature reactions and sparking. Illustrated. 8vo., cloth, 295 pp. . . . . .Net, $3.00

WILKINSON, H. D. **Submarine Cable-Laying, Repairing, and Testing.** *New Edition.* Illustrated. 8vo., cloth. . . . .*In Press*

YOUNG, J. ELTON. **Electrical Testing for Telegraph Engineers.** Illustrated. 8vo., cloth, 264 pp. . . . . .Net, $4.00

ZEIDLER, J. and LUSTGARTEN, J. **Electric Arc Lamps.** Their principles, construction and working. 160 illustrations. 8vo., cloth, 200 pp. . . . .Net, $2.00